

Leigh R. Hochberg et al.

# Reach and grasp by people with tetraplegia using a neurally controlled robotic arm

Nature, 17 May 2012

Paper overview

Ilya Kuzovkin

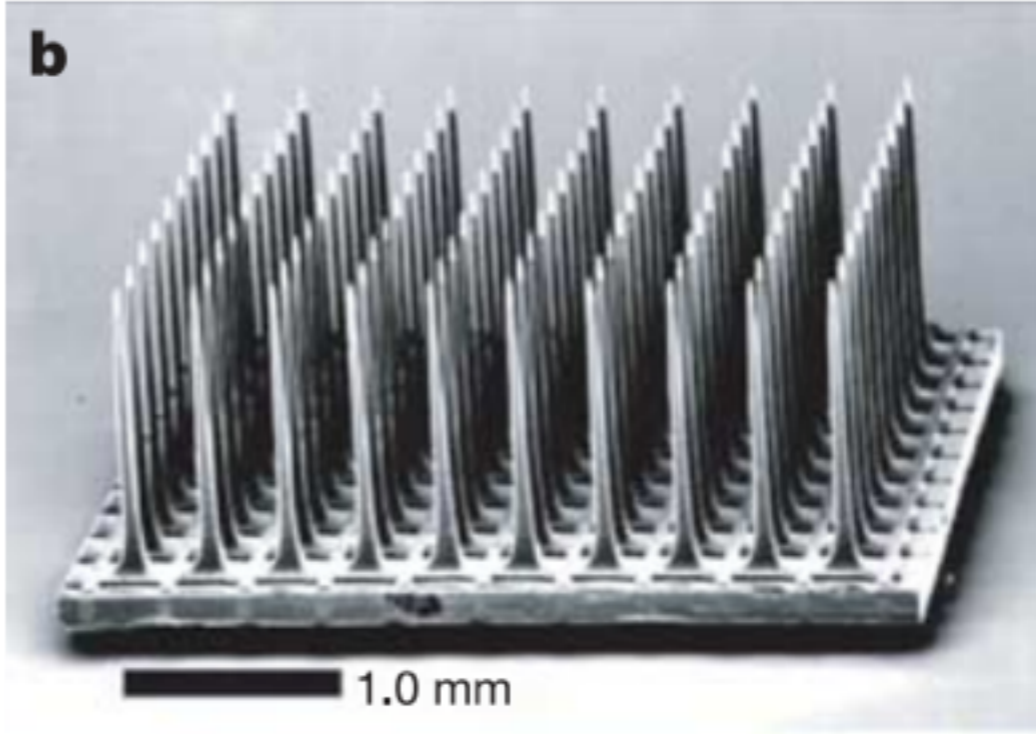
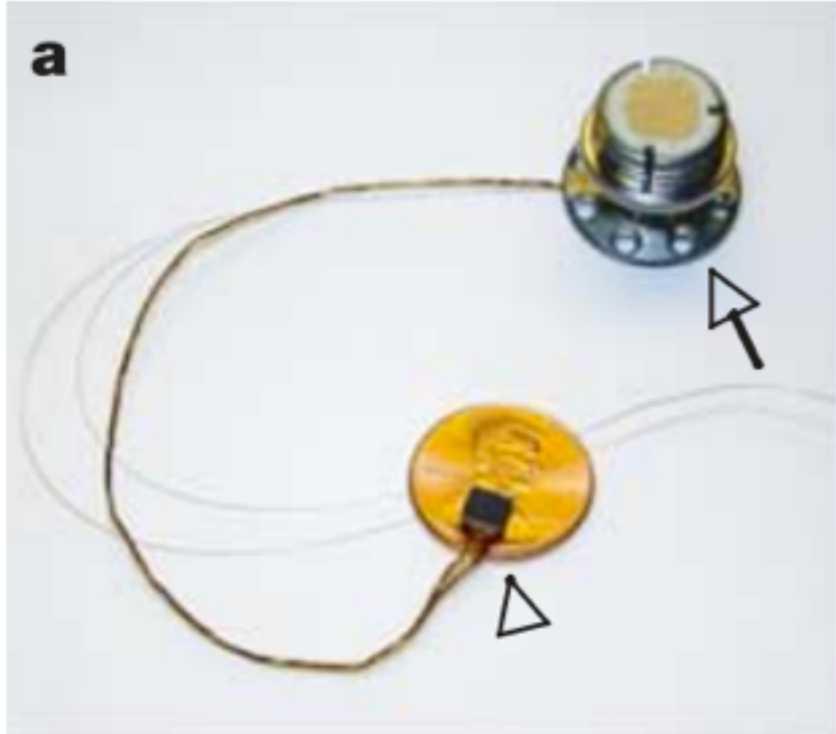


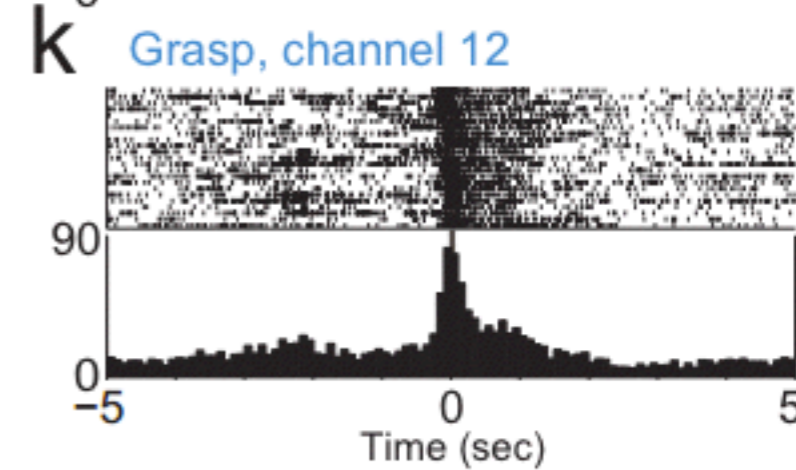
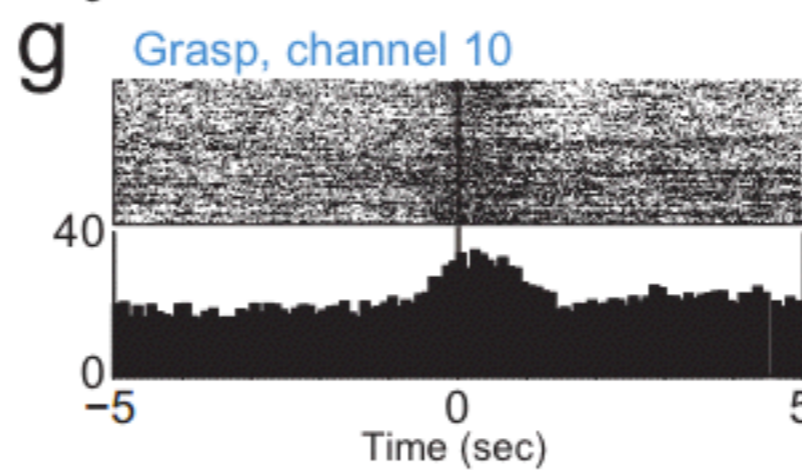
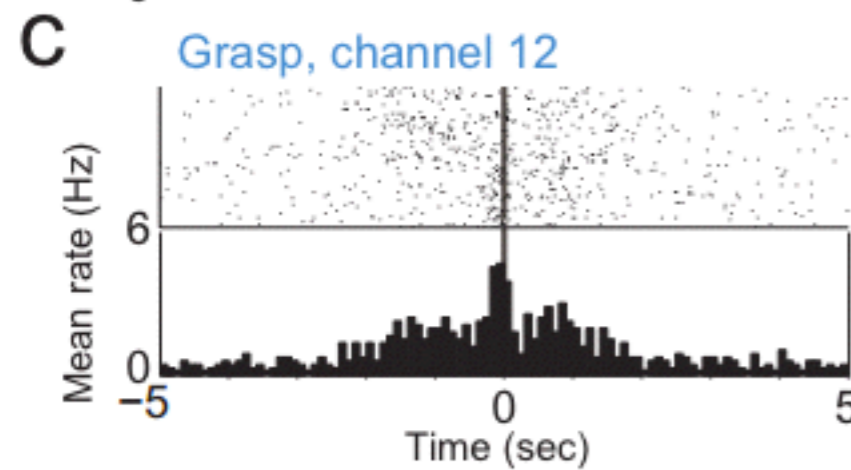
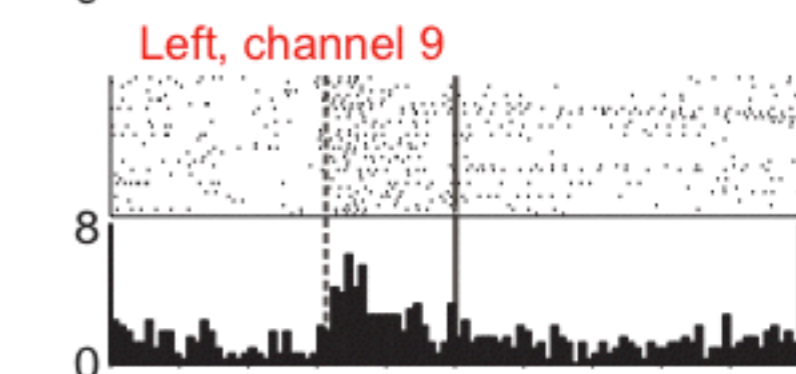
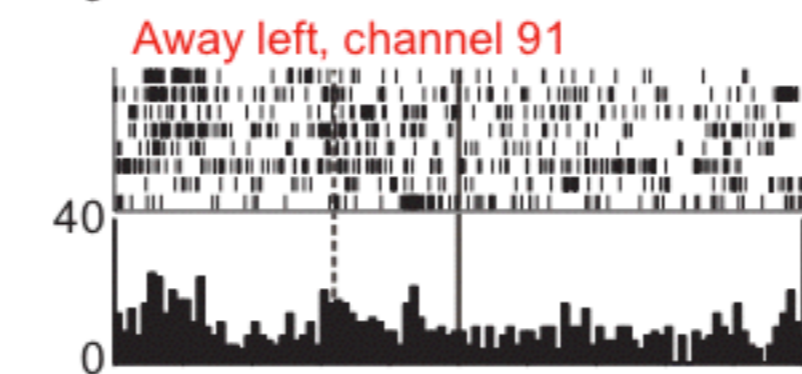
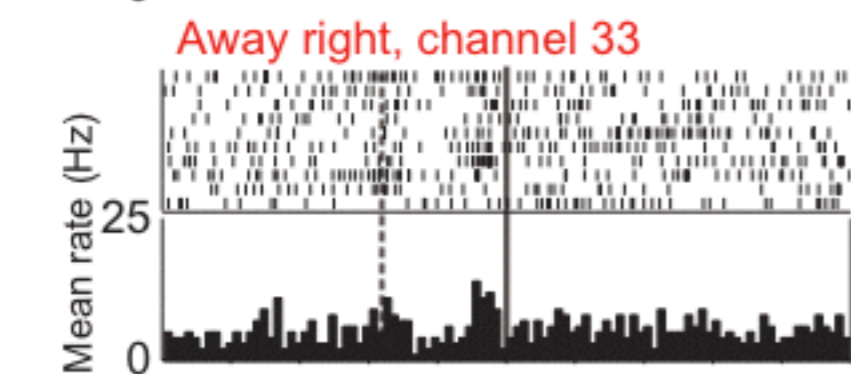
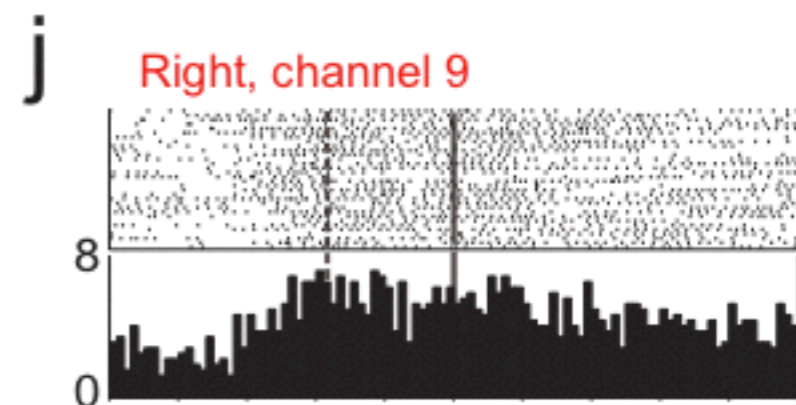
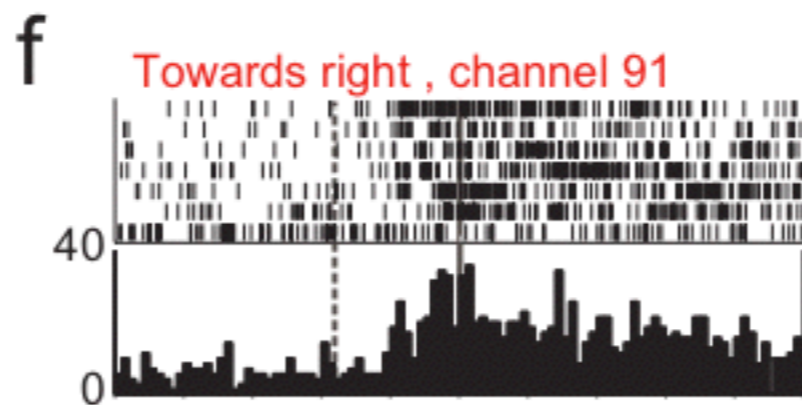
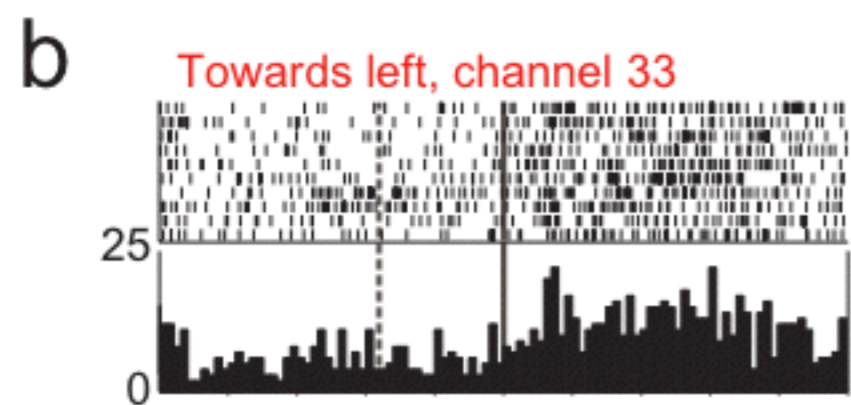
11 April 2014, Tartu

BrainGate Pilot Clinical Trial  
Drinking From a Bottle Using a Robotic Arm  
Participant S3  
Trial Day 1959 / 12 April 2011  
Hochberg *et al.*, 2012

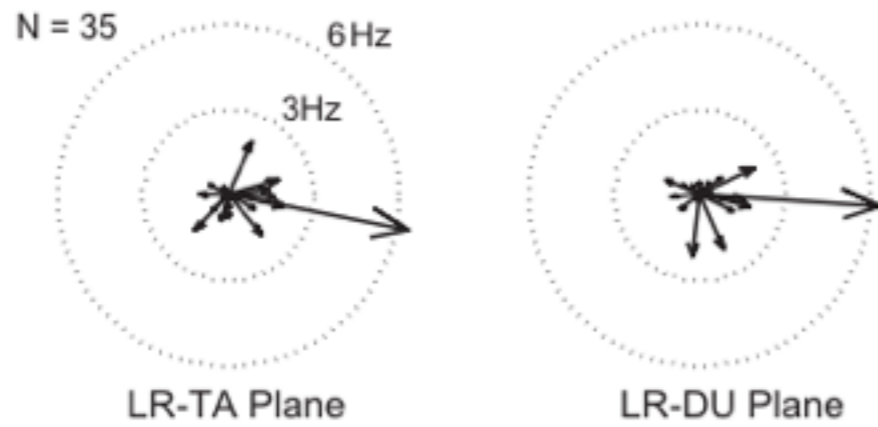


Caution: Investigational Device. Limited by Federal Law to Investigational Use.

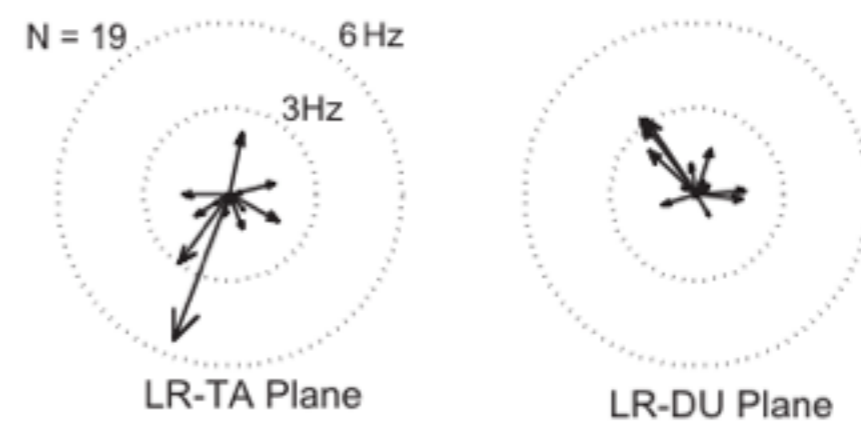




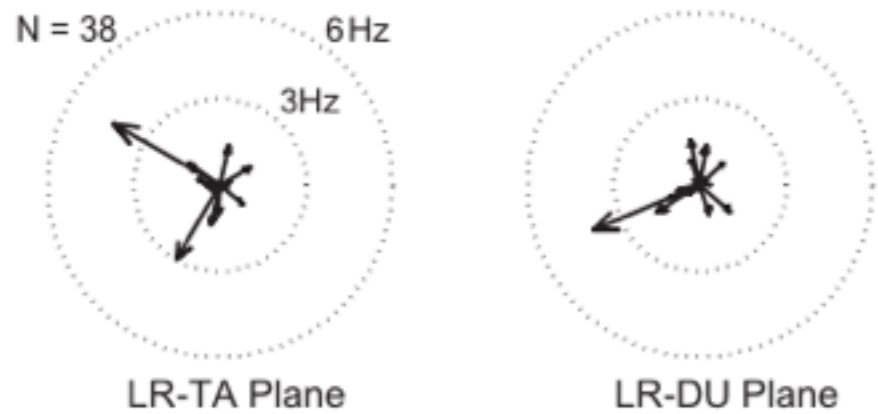
**a** S3: DLR 3D Task  
Trial Day 1952



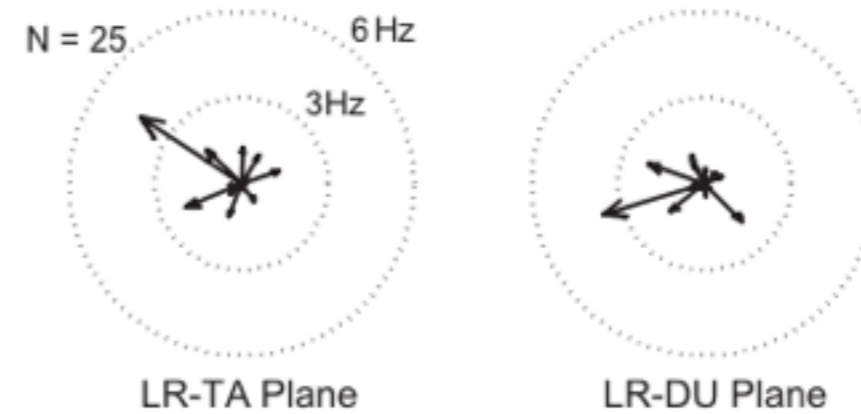
**b** S3: DLR 3D Task  
Trial Day 1959



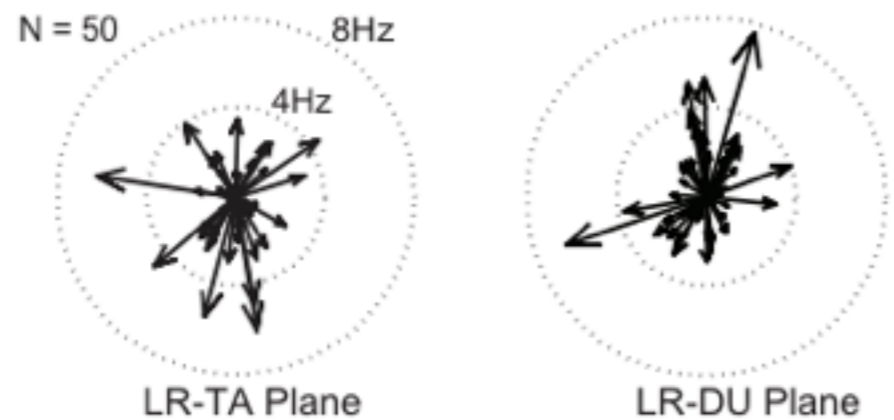
**c** S3: DEKA 3D Task  
Trial Day 1974



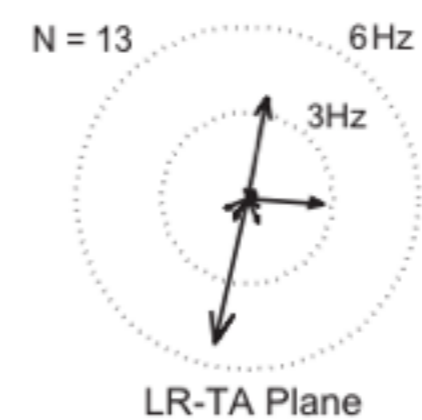
**d** S3: DEKA 3D Task  
Trial Day 1975

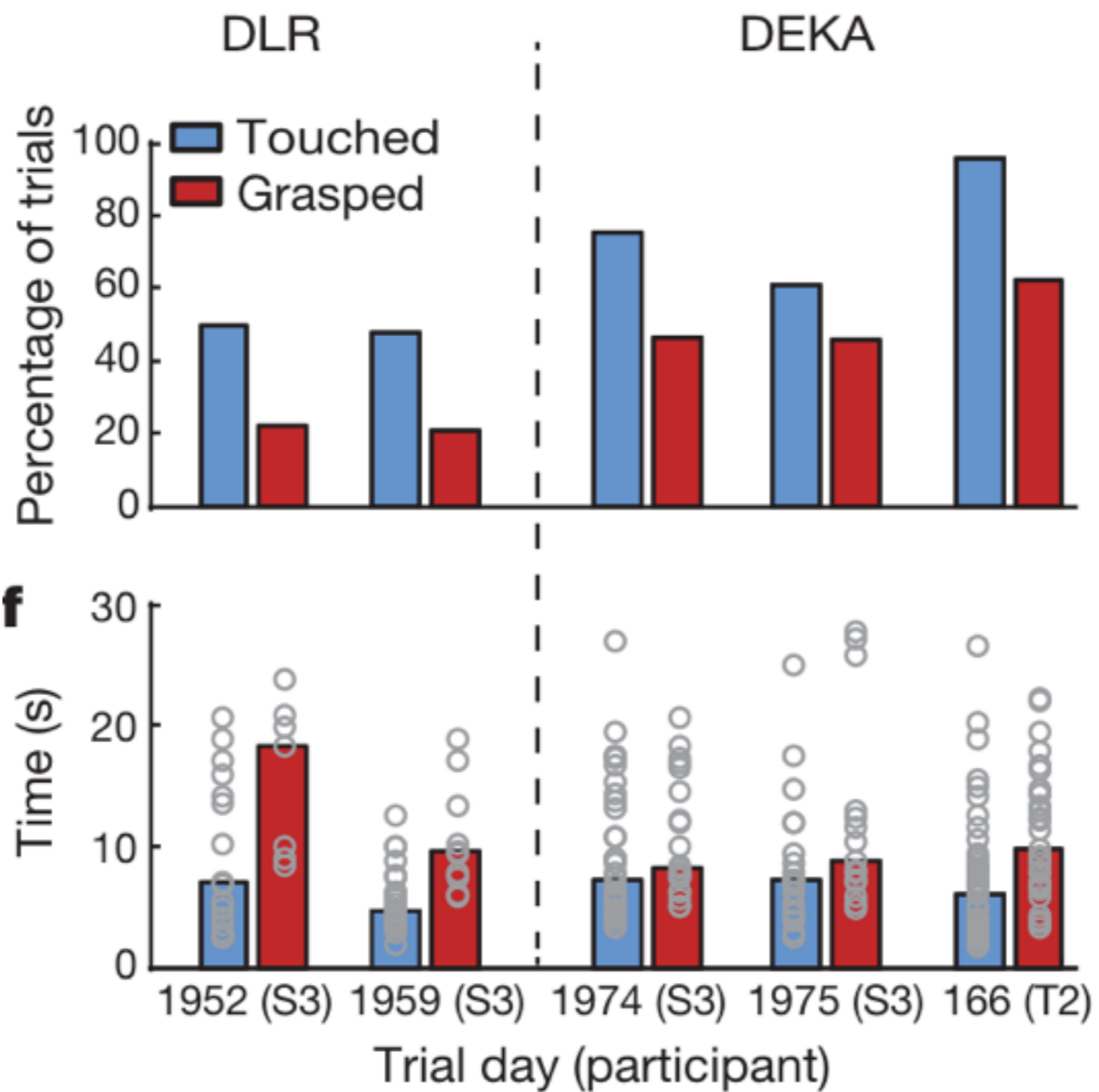
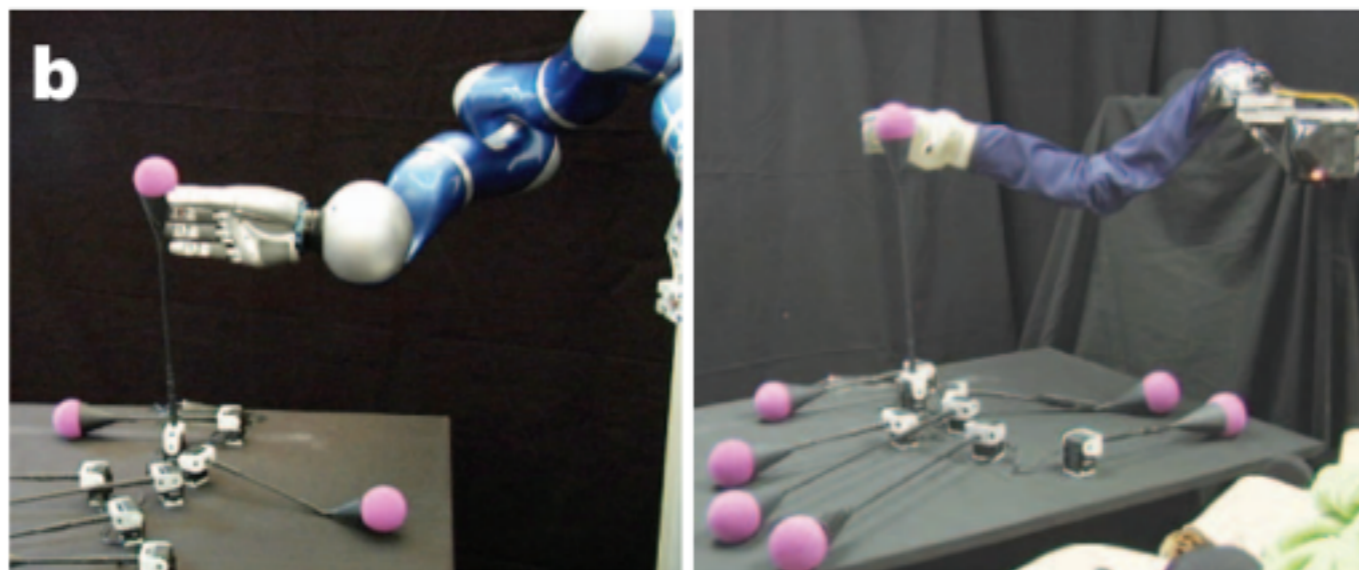


**e** T2: DEKA 3D Task  
Trial Day 166



**f** S3: DLR Drinking demonstration  
Trial Day 1959





adapted to the Kalman filter from refs 37 and 38:

desired endpoint velocity was decoded using a Kalman filter<sup>7,8,12,36</sup>. The Kalman filter requires four sets of parameters, two of which were calculated based on the mean-subtracted (and for T2, smoothed with a 0.3 s exponential filter) threshold crossing rate,  $\bar{z}$ , and the intended direction,  $d$ , whereas the other two parameters were hard coded. The first parameter was the directional tuning,  $H$ , calculated as

etc...

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How it works?

etc...



2012

Reach and grasp by people with tetraplegia using a neurally controlled robotic arm

Efficient Decoding With Steady-State Kalman Filter in Neural Interface Systems

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Neural control of cursor trajectory and click by a human with tetraplegia 1000 days after implant of an intracortical microelectrode array

Point-and-Click Cursor Control With an Intracortical Neural Interface System by Humans With Tetraplegia

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Neuronal ensemble control of prosthetic devices by a human with tetraplegia

Bayesian Population Decoding of Motor Cortical Activity using a Kalman Filter

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*“... uses Bayesian inference techniques to estimate hand motion from the firing rates of multiple neurons.”*

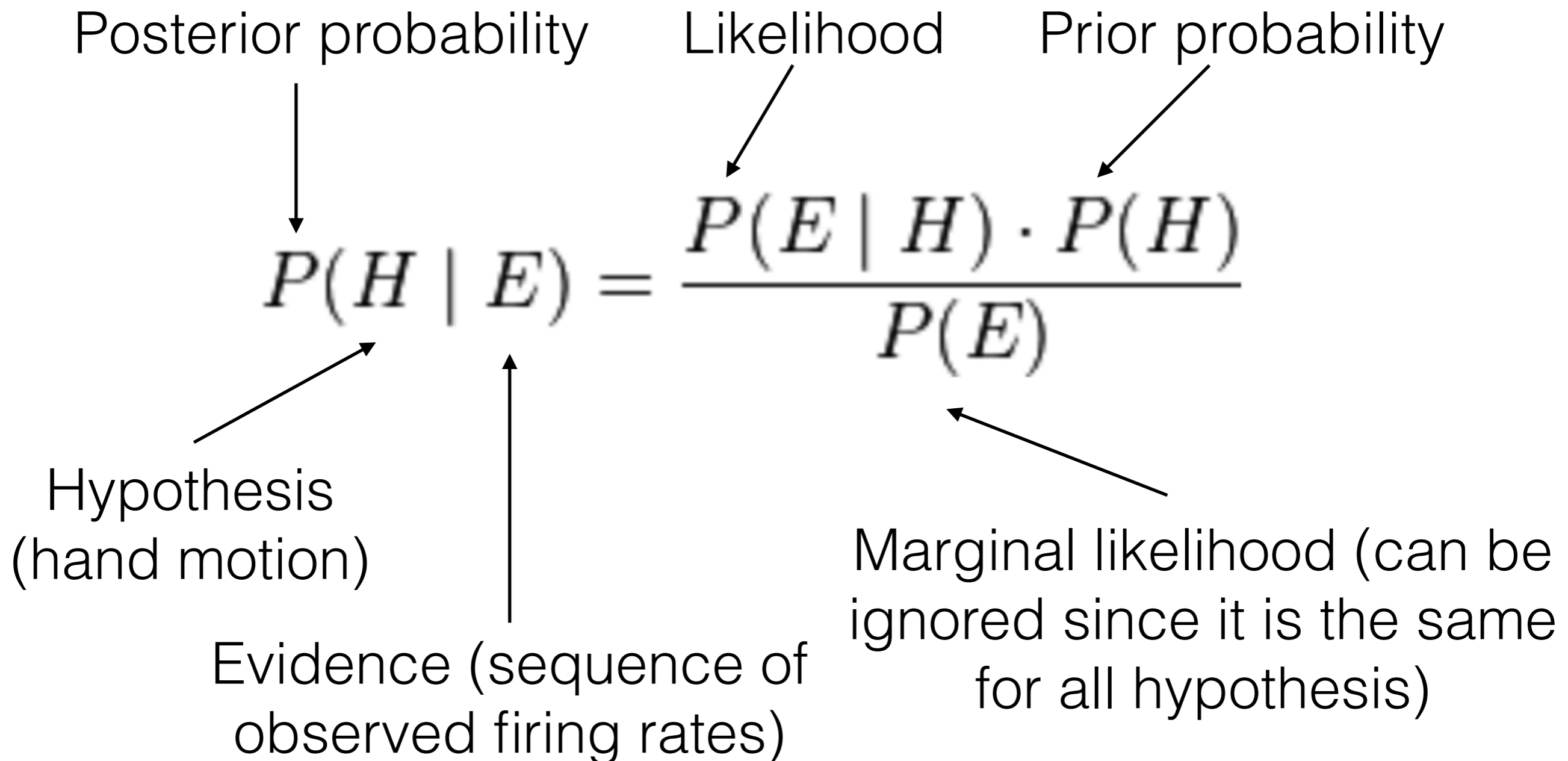
*“Decoding was performed using a Kalman filter which gives an efficient recursive method for Bayesian inference ...”*

*“... uses **Bayesian inference** techniques to estimate hand motion from the firing rates of multiple neurons.”*

*“... uses Bayesian inference techniques to estimate hand motion from the firing rates of multiple neurons.”*

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Posterior probability      Likelihood      Prior probability

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Hypothesis (hand motion)      Evidence (sequence of observed firing rates)      Marginal likelihood (can be ignored since it is the same for all hypothesis)

The diagram illustrates Bayes' theorem with the following components and labels:

- Posterior probability:**  $P(H | E)$
- Likelihood:**  $P(E | H)$
- Prior probability:**  $P(H)$
- Marginal likelihood:**  $P(E)$

Labels and arrows indicate the following relationships:

- An arrow points from "Posterior probability" to  $P(H | E)$ .
- An arrow points from "Likelihood" to  $P(E | H)$ .
- An arrow points from "Prior probability" to  $P(H)$ .
- An arrow points from "Hypothesis (hand motion)" to  $H$  in  $P(H | E)$ .
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*“... uses Bayesian inference techniques to estimate hand motion from the firing rates of multiple neurons.”*

*“Likelihood term models the probability of firing rates given a particular hand motion”*

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The diagram shows the equation for Bayes' theorem. Arrows point from labels to terms in the equation: 'Posterior probability' points to  $P(H | E)$ ; 'Likelihood' points to  $P(E | H)$ ; 'Prior probability' points to  $P(H)$ ; 'Hypothesis (hand motion)' points to  $H$ ; 'Evidence (sequence of observed firing rates)' points to  $E$ ; and 'Marginal likelihood (can be ignored since it is the same for all hypothesis)' points to  $P(E)$ .

*“The prior term defines a probabilistic model of hand kinematics and was also taken to be a linear Gaussian model.”*

# Neural Coding

# Neural Coding of Hand Kinematics

Georgopoulos *et al.* (1982) observed that the firing rate of cells in MI were approximated by a cosine “tuning function.” In particular the firing rate,  $z_k$ , of a cell at some time  $t_k$  is related to movement *direction*,  $\theta_k$ , by

$$z_k = h_0 + h_p \cos(\theta_k - \theta_p) \quad (1)$$

where  $\theta_p$  is the cell’s “preferred” direction (i.e. direction of maximal response) and  $h_0$  and  $h_p$  are scalar constants. Equivalently, this can be expressed as

$$z_k = h_0 + h_1 \sin(\theta_k) + h_2 \cos(\theta_k), \quad (2)$$

where  $h_1, h_2$  are scalar parameters that can be fit to training data.

# Neural Coding of Hand Kinematics

Moran and Schwartz (Moran and Schwartz, 1999b) extended the above model to include the speed,  $\|v\|$ , of the hand

$$z_k = h_0 + \|v\|(h_x \sin(\theta_k) + h_y \cos(\theta_k)). \quad (3)$$

This is equivalent to modeling the firing rate as a linear function of velocity in the  $x$  and  $y$  coordinates:

$$z_k = h_0 + h_x v_{x,k} + h_y v_{y,k}, \quad (4)$$

where  $v_{x,k}$  and  $v_{y,k}$  represent the hand velocity in the  $x$  and  $y$  directions respectively at time  $t_k$ .

# Neural Coding of Hand Kinematics

A similar linear model was proposed to relate firing rate and hand position (Kettner et al., 1988):

$$z_k = f_0 + f_x x_k + f_y y_k, \quad (5)$$

where  $x_k$  and  $y_k$  represent hand position and  $f_0, f_x, f_y$  are the linear coefficients which are fit to training data.

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Experiment 1: 23/25 neurons are correctly described by equations (4) and (5)

Experiment 2: 39/42 neurons correctly described by (4) and (5)

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ts which

The relationship between the kinematics of the arm and the behavior of the neurons is strong

described by equations (4) and (5)

Experiment 2: 39/42 neurons correctly described by (4) and (5)



Learning the model

# Detour: Multivariate normal distribution

The multivariate normal distribution of a  $k$ -dimensional random vector  $\mathbf{x} = [X_1, X_2, \dots, X_k]$

$$\mathbf{x} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}),$$

or to make it explicitly known that  $X$  is  $k$ -dimensional,

$$\mathbf{x} \sim \mathcal{N}_k(\boldsymbol{\mu}, \boldsymbol{\Sigma}).$$

with  $k$ -dimensional **mean vector**

$$\boldsymbol{\mu} = [E[X_1], E[X_2], \dots, E[X_k]]$$

and  $k \times k$  **covariance matrix**

$$\boldsymbol{\Sigma} = [\text{Cov}[X_i, X_j]], i = 1, 2, \dots, k; j = 1, 2, \dots, k$$

Every linear combination of its components  $Y = a_1X_1 + \dots + a_kX_k$  is **normally distributed**. That is, for any constant vector  $\mathbf{a} \in \mathbf{R}^k$ , the random variable  $Y = \mathbf{a}'\mathbf{x}$  has a univariate normal distribution.

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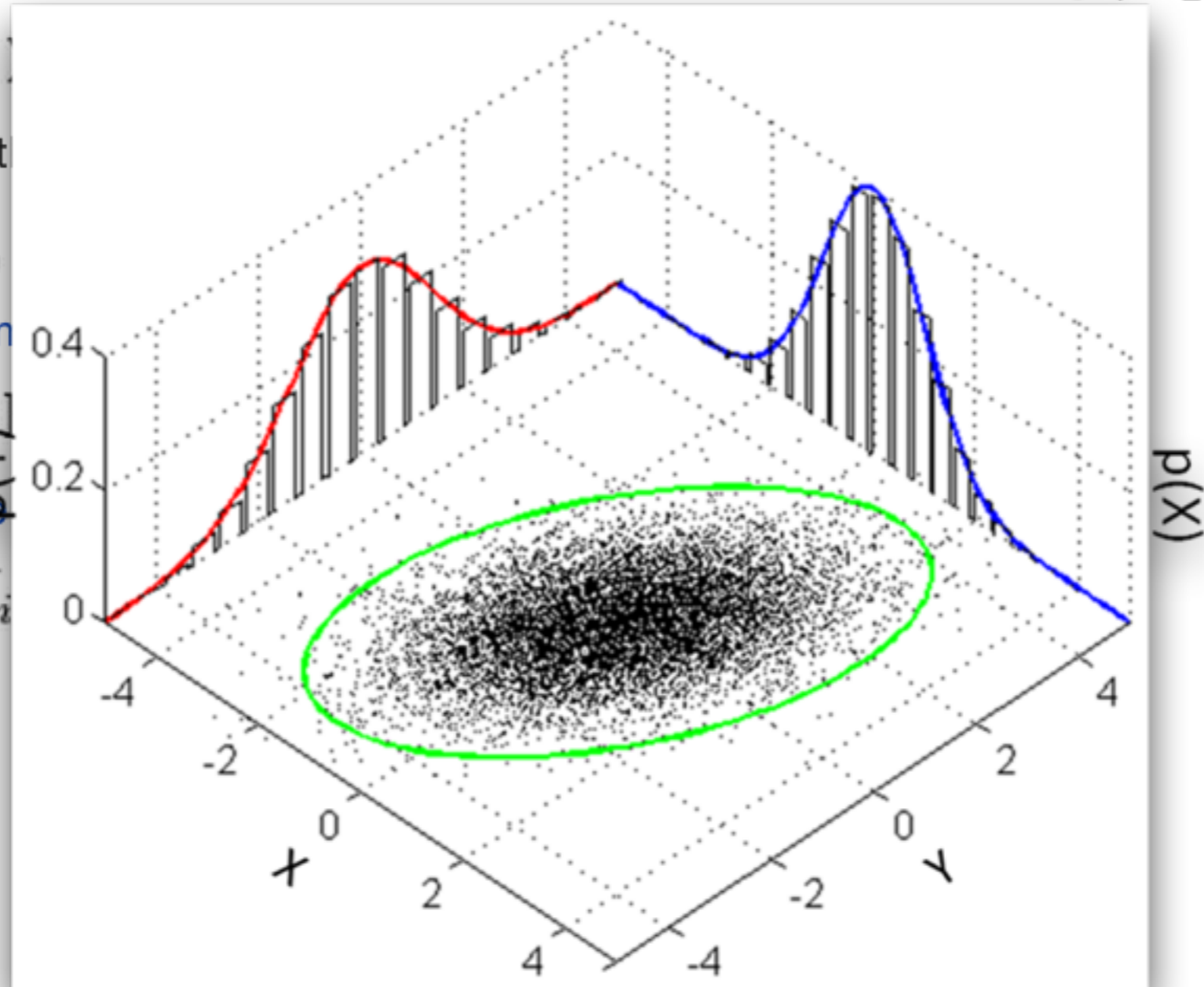
$$\mathbf{x} \sim \mathcal{N}_k(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

with  $k$ -dimensional mean

$$\boldsymbol{\mu} = [E[X_1], \dots, E[X_k]]$$

and  $k \times k$  covariance

$$\boldsymbol{\Sigma} = [\text{Cov}[X_i, X_j]]$$



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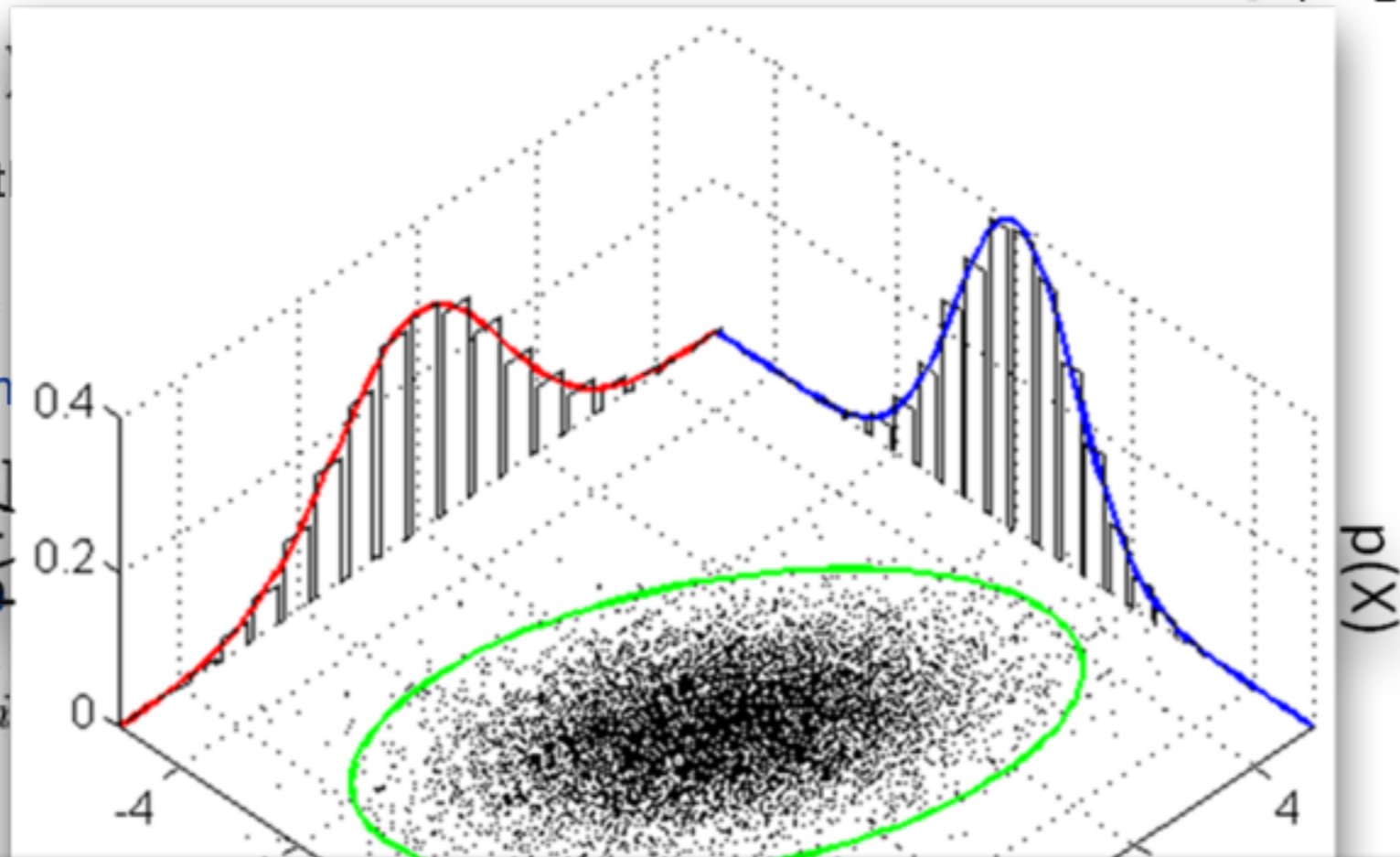
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Why covariance matrix and not just a vector of variances?

Every linear combination of its components  $Y = a_1X_1 + \dots + a_kX_k$  is **normally distributed**. That is, for any constant vector  $\mathbf{a} \in \mathbf{R}^k$ , the random variable  $Y = \mathbf{a}'\mathbf{x}$  has a univariate normal distribution.

# Definitions

We then define the system *state* to be a six-dimensional vector  $\mathbf{x}_k = [x, y, v_x, v_y, a_x, a_y]_k^T$  representing the  $x$ -position,  $y$ -position,  $x$ -velocity,  $y$ -velocity,  $x$ -acceleration, and  $y$ -acceleration of the hand at time  $t_k = k\Delta t$  where  $\Delta t = 70ms$

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let observations  $\mathbf{z}_k \in \mathbb{R}^C$  represent a  $C \times 1$  vector containing the firing rates at time  $t_k$  for  $C$  observed neurons within a  $\Delta t$  time interval. Let  $\mathbf{Z}_k = [\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_k]^T$  represent the history of measurements up to time bin  $k$ .

# Parameters of the model

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$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{q}_k,$$

$$\mathbf{q}_k \sim N(0, \mathbf{Q}_k), \mathbf{Q}_k \in \mathbb{R}^{C \times C}$$

**H** is the relation between the firing rates of each of the neurons and states of the arm

**Q** is covariance matrix of the noise



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$$\mathbf{x}_{k+1} = \mathbf{A}_k \mathbf{x}_k + \mathbf{w}_k,$$

$$\mathbf{w}_k \sim N(0, \mathbf{W}_k), \mathbf{W}_k \in \mathbb{R}^{6 \times 6}$$

**A** is the relation between the state at time  $k+1$  and the state at time  $k$

**W** is covariance matrix of the noise

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Matrices  $\mathbf{A}$ ,  $\mathbf{H}$ ,  $\mathbf{Q}$ ,  $\mathbf{W}$  is what we want to learn from the training data

# The Learning

we can estimate all the constant parameters  $\mathbf{A}$ ,  $\mathbf{H}$ ,  $\mathbf{W}$ ,  $\mathbf{Q}$  from training data by maximizing the joint probability  $p(\mathbf{X}_M, \mathbf{Z}_M)$ , where both the hand kinematics  $\mathbf{X}_M = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_M]^T$  and the firing rates  $\mathbf{Z}_M$  are known for the  $M$  time instants in the training set.

$$p(\mathbf{X}_M, \mathbf{Z}_M) = [p(\mathbf{x}_1) \prod_{k=2}^M p(\mathbf{x}_k | \mathbf{x}_{k-1})] [\prod_{k=1}^M p(\mathbf{z}_k | \mathbf{x}_k)].$$

This is simply the product of the prior hand state  $p(\mathbf{x}_1)$  at the first time instant, the probability of each new state conditioned on the previous one, and the likelihood at each time instant. Given training data we maximize this probability with respect to  $\mathbf{A}$ ,  $\mathbf{H}$ ,  $\mathbf{W}$ ,  $\mathbf{Q}$

Decoding

*“Decoding was performed using a Kalman filter which gives an efficient recursive method for Bayesian inference ...”*

We pose the problem of estimating the system state as one of Bayesian inference.

Let  $p(\mathbf{x}_k | \mathbf{Z}_k)$  be the *a posteriori* probability of the system state conditioned on the measurements.

Posterior probability

Likelihood

Prior probability

$$P(H | E) = \frac{P(E | H) \cdot P(H)}{P(E)}$$

Hypothesis (hand motion)

Evidence (sequence of observed firing rates)

Marginal likelihood (can be ignored since it is the same)

The diagram shows the equation  $P(H | E) = \frac{P(E | H) \cdot P(H)}{P(E)}$  with arrows pointing from descriptive labels to the corresponding parts of the equation. 'Posterior probability' points to  $P(H | E)$ . 'Likelihood' points to  $P(E | H)$ . 'Prior probability' points to  $P(H)$ . 'Hypothesis (hand motion)' points to  $H$ . 'Evidence (sequence of observed firing rates)' points to  $E$ . 'Marginal likelihood (can be ignored since it is the same)' points to  $P(E)$ .

Note that now  $\mathbf{x}$  and  $\mathbf{z}$  and everything else refer to the test data

*“Decoding was performed using a Kalman filter which gives an efficient recursive method for Bayesian inference ...”*

Then, using Bayes Theorem and simple algebra we can write the posterior probability

$$p(\mathbf{x}_k | \mathbf{Z}_k) = \kappa p(\mathbf{z}_k | \mathbf{x}_k) \int p(\mathbf{x}_k | \mathbf{x}_{k-1}) p(\mathbf{x}_{k-1} | \mathbf{Z}_{k-1}) d\mathbf{x}_{k-1}, \quad (8)$$

where  $p(\mathbf{z}_k | \mathbf{x}_k)$  is called the *likelihood* of the state,  $p(\mathbf{x}_k | \mathbf{x}_{k-1})$  is a *temporal prior* that models how the state evolves from one time instant to the next, and  $p(\mathbf{x}_{k-1} | \mathbf{Z}_{k-1})$  is simply the posterior probability at the previous time instant. The term  $\kappa$  is a normalizing term, independent of  $\mathbf{x}_k$ , which ensures that posterior integrates to 1.

$$\begin{array}{c}
 \text{Posterior probability} \\
 \downarrow \\
 P(H | E) = \frac{P(E | H) \cdot P(H)}{P(E)} \\
 \begin{array}{l}
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 \end{array}
 \end{array}
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 \nearrow \\
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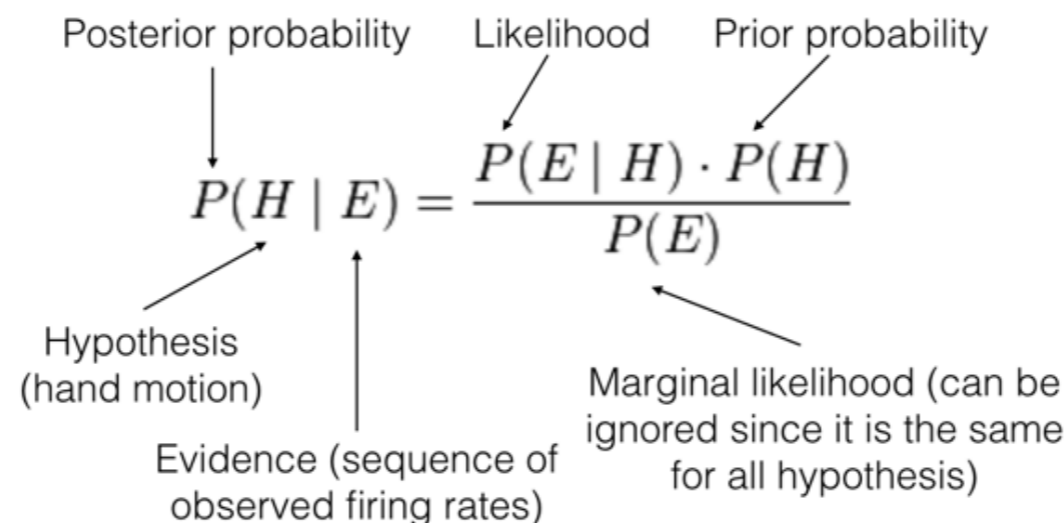
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The probability that the hand can move in the way it did



“Decoding was performed using a Kalman filter which gives an efficient recursive method for Bayesian inference ...”

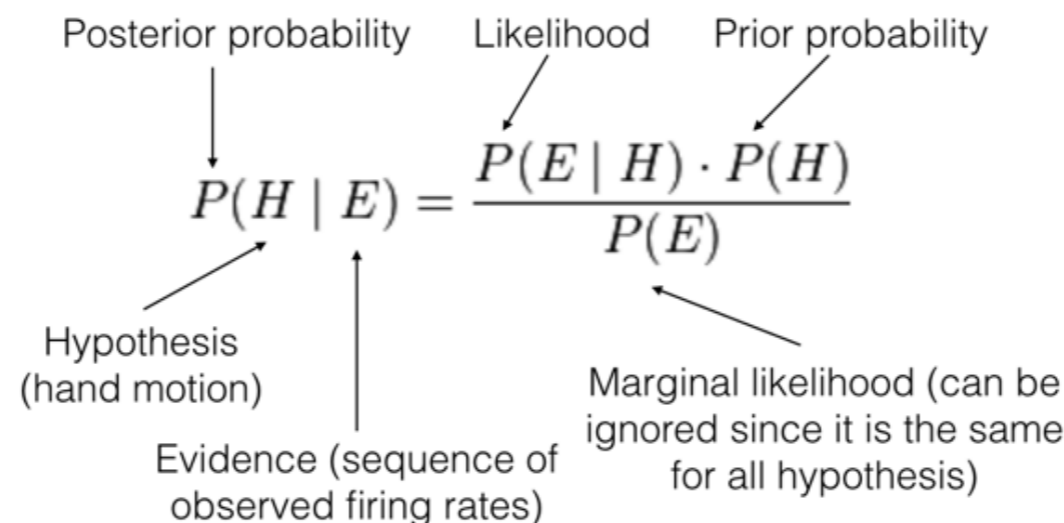
Then, using Bayes Theorem and simple algebra we can write the posterior probability

$$p(\mathbf{x}_k | \mathbf{Z}_k) = \kappa p(\mathbf{z}_k | \mathbf{x}_k) \int p(\mathbf{x}_k | \mathbf{x}_{k-1}) p(\mathbf{x}_{k-1} | \mathbf{Z}_{k-1}) d\mathbf{x}_{k-1}, \quad (8)$$

where  $p(\mathbf{z}_k | \mathbf{x}_k)$  is called the *likelihood* of the state,  $p(\mathbf{x}_k | \mathbf{x}_{k-1})$  is a *transitional prior* that models how the state evolves from one time instant to the next, and  $p(\mathbf{x}_{k-1} | \mathbf{Z}_{k-1})$  is simply the posterior probability of the state at time  $k-1$ . The term  $\kappa$  is a normalization term, independent of  $\mathbf{x}_k$ , which ensures that posterior integrals sum to one.

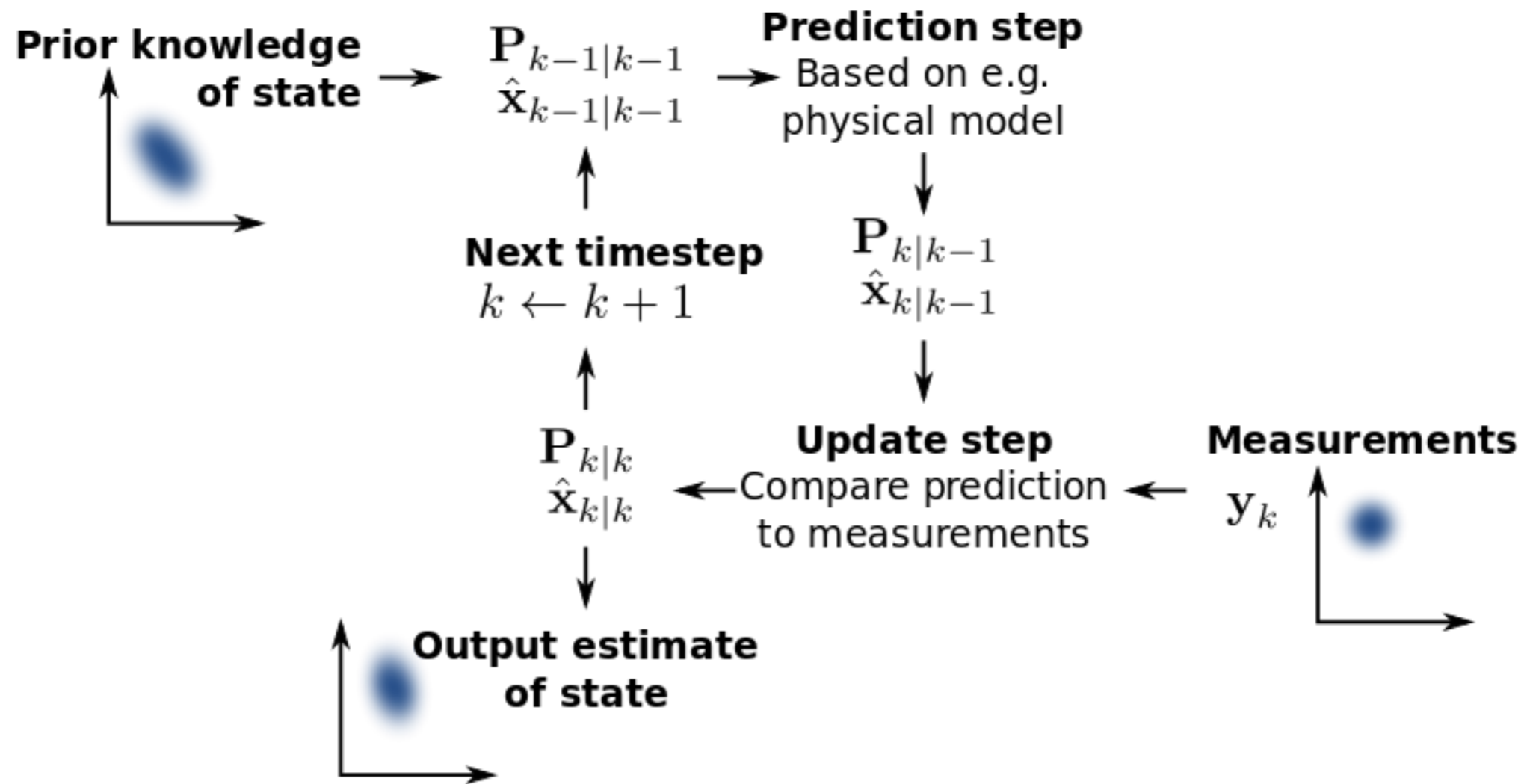
The probability that the hand can move in the way it did

The probability that hand can end up in the state where it was in time k-1





*“Decoding was performed using a **Kalman filter** which gives an efficient recursive method for **Bayesian inference** ...”*



*“... the Kalman filter operates recursively on streams of noisy input data to produce a statistically optimal estimate of the underlying system state.” (Wikipedia)*

*“Decoding was performed using a Kalman filter which gives an efficient recursive method for Bayesian inference ...”*

*“Decoding was performed using a Kalman filter which gives an efficient recursive method for Bayesian inference ...”*

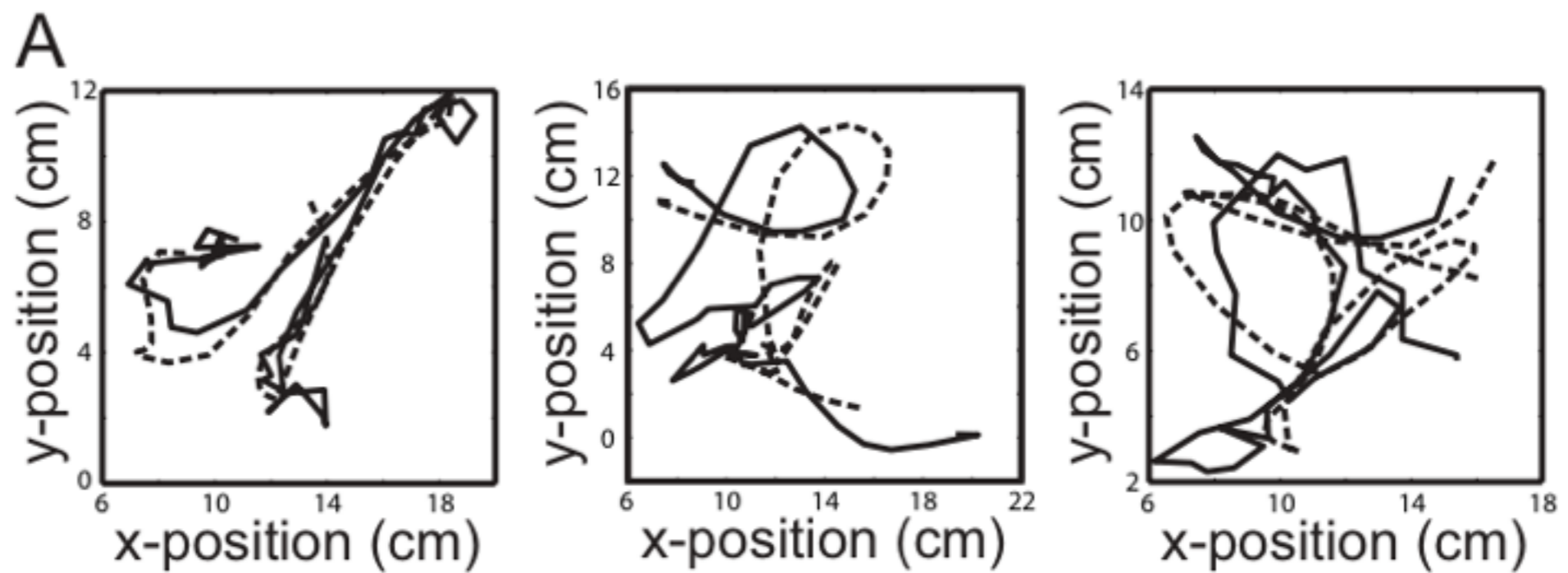
Decoding then involves estimating the posterior probability,  $p(\mathbf{x}_k | \mathbf{Z}_k)$ , at each time instant, which we can do recursively using Eq. (8).

*“Decoding was performed using a Kalman filter which gives an efficient recursive method for Bayesian inference ...”*

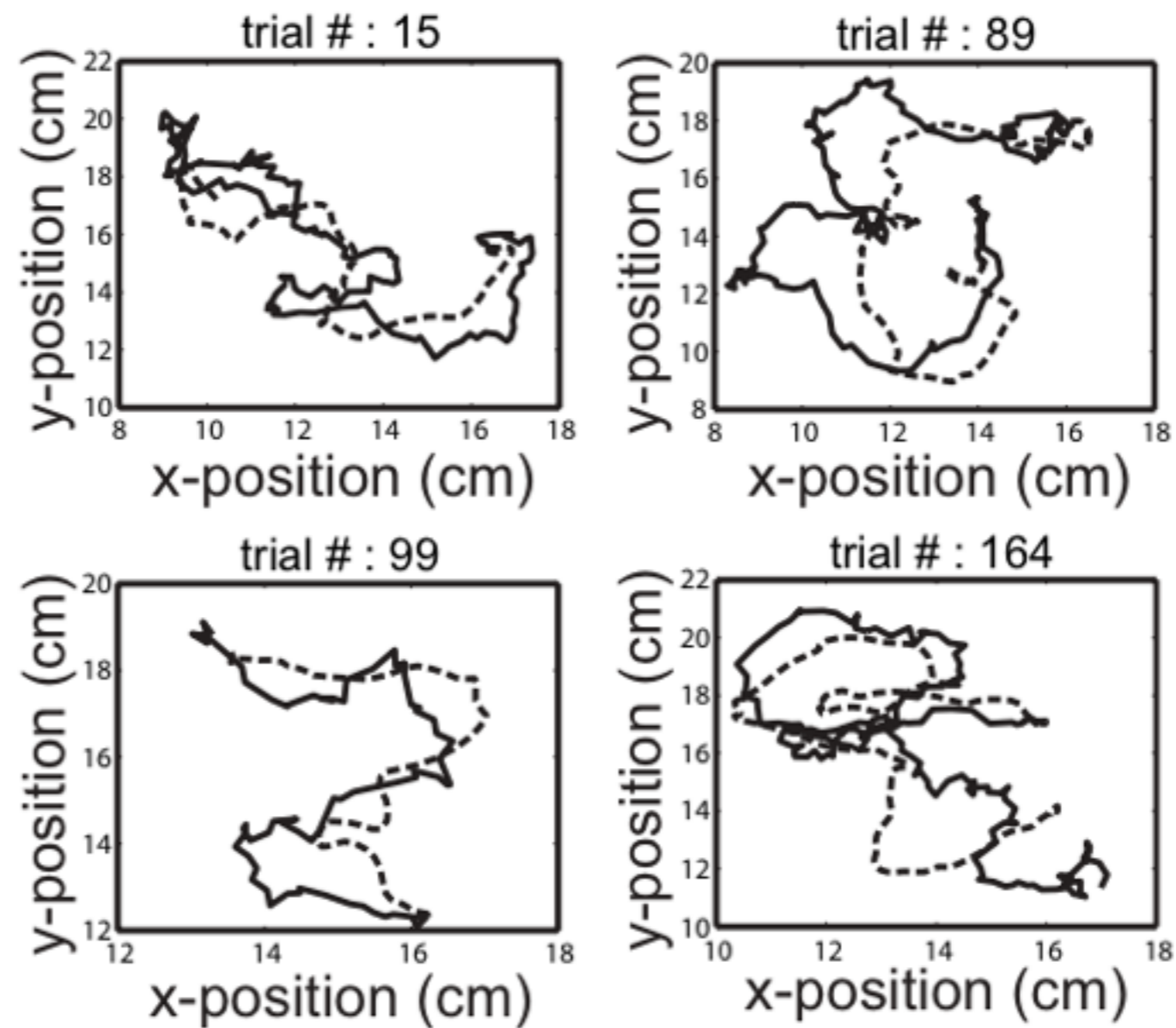
Decoding then involves estimating the posterior probability,  $p(\mathbf{x}_k | \mathbf{Z}_k)$ , at each time instant, which we can do recursively using Eq. (8).

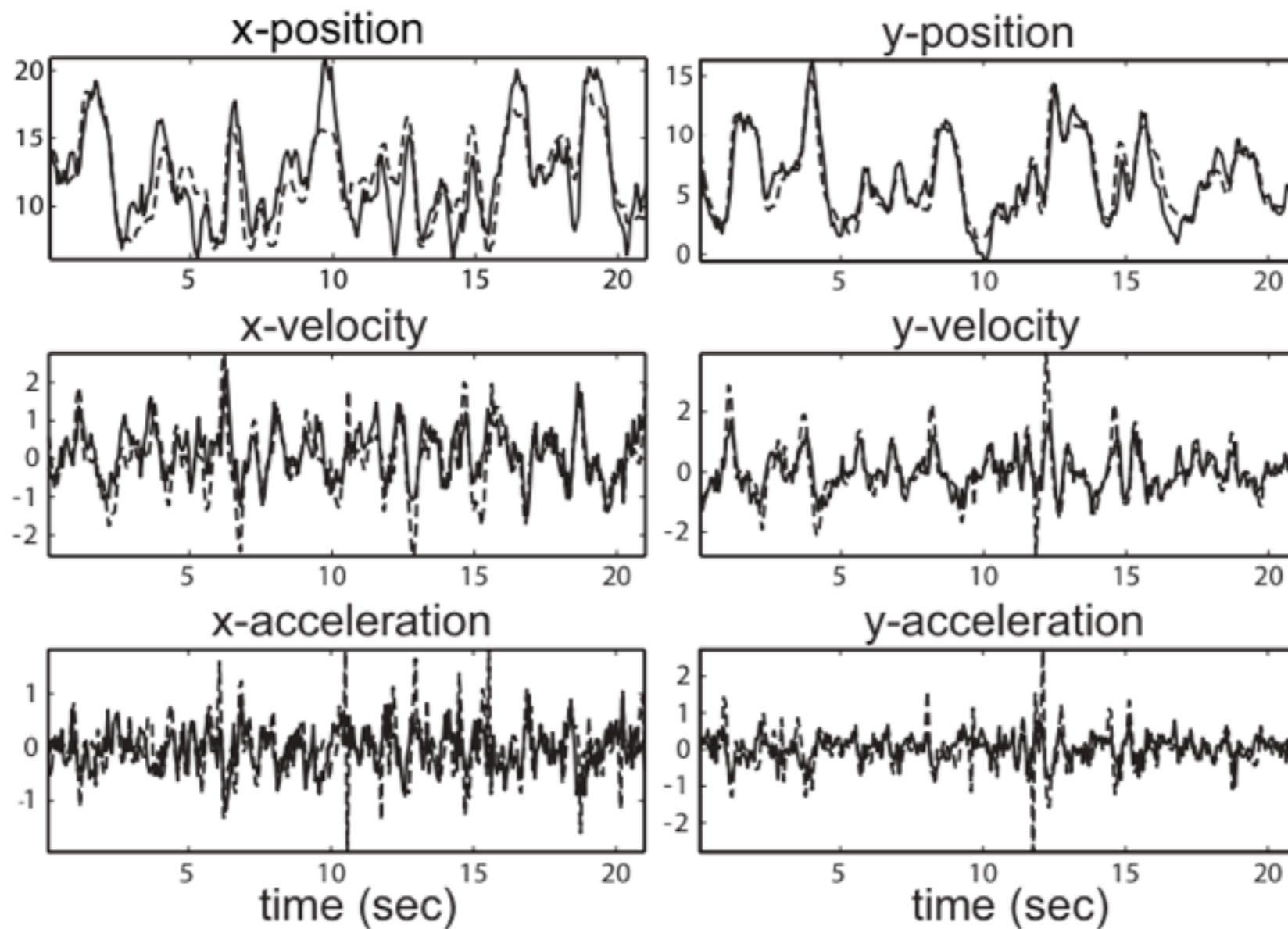
To achieve a real-time decoding algorithm, we assume that the likelihood and the prior are both Gaussian, and this leads to a closed-form recursive solution for estimating the posterior (which is also Gaussian); this is known as the Kalman filter (Kalman, 1960; Gelb, 1974; Welch and Bishop, 2001).

# Results



**B**





Method	$CC(x, y)$	$MSE (cm^2)$
Kalman (0ms lag)	(0.78, 0.91)	7.01
Kalman (70ms lag)	(0.80, 0.93)	6.25
<b>Kalman (140ms lag)</b>	<b>(0.82, 0.93)</b>	<b>5.87</b>
Kalman (210ms lag)	(0.81, 0.89)	6.80
Kalman (280ms lag)	(0.76, 0.82)	8.81
<b>Kalman (non-uni lag, init 1)</b>	<b>(0.84, 0.93)</b>	<b>4.75</b>
Kalman (non-uni lag, init 2)	(0.84, 0.93)	4.77

2012

Reach and grasp by people with tetraplegia using a neurally controlled robotic arm

Efficient Decoding With Steady-State Kalman Filter in Neural Interface Systems

2011

Neural control of cursor trajectory and click by a human with tetraplegia 1000 days after implant of an intracortical microelectrode array

Point-and-Click Cursor Control With an Intracortical Neural Interface System by Humans With Tetraplegia

2010

Neural control of computer cursor velocity by decoding motor cortical spiking activity in humans with tetraplegia

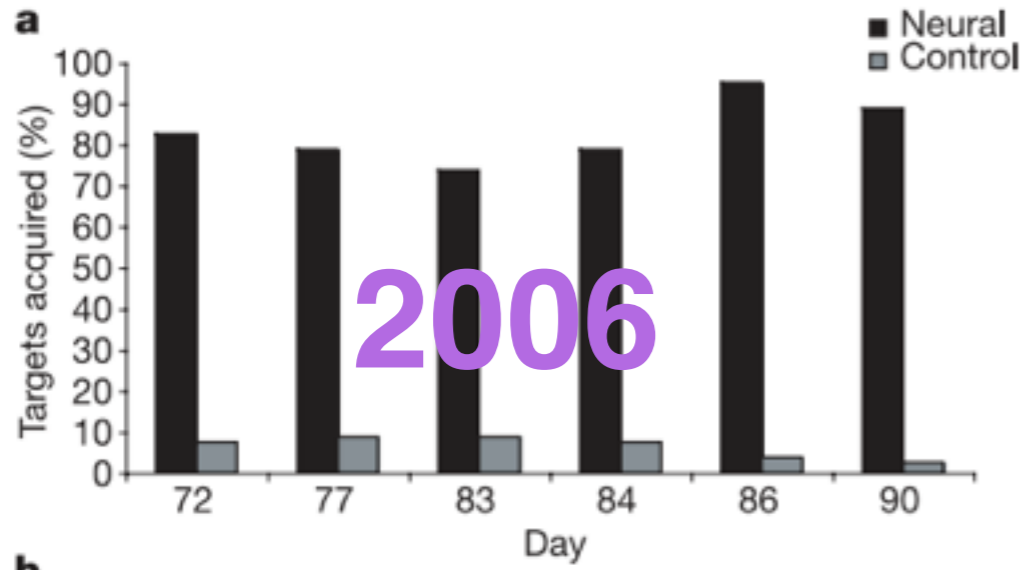
2006

Neuronal ensemble control of prosthetic devices by a human with tetraplegia

Bayesian Population Decoding of Motor Cortical Activity using a Kalman Filter

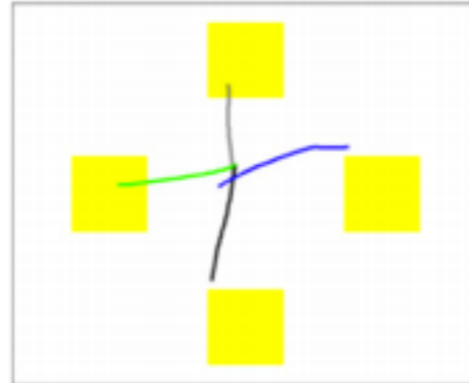
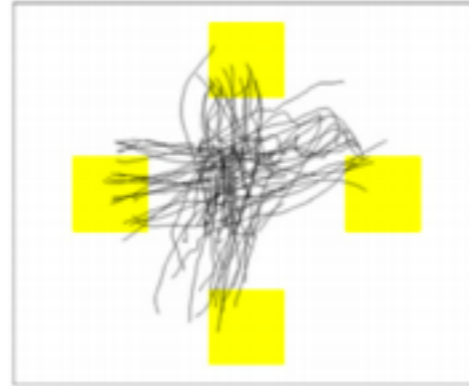




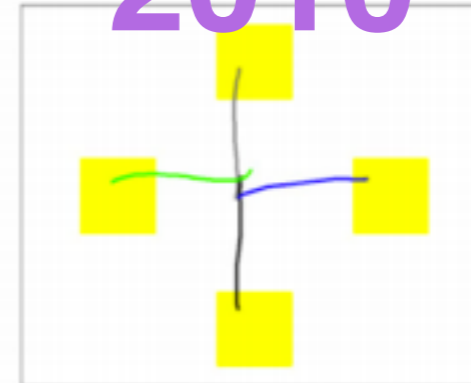
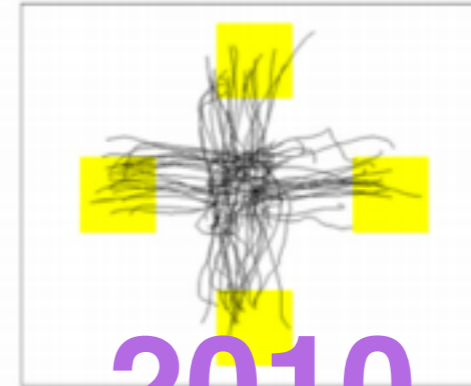


2006

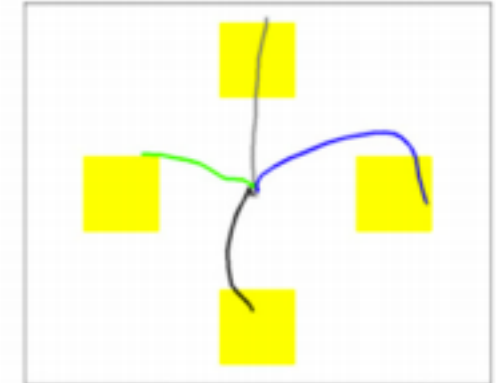
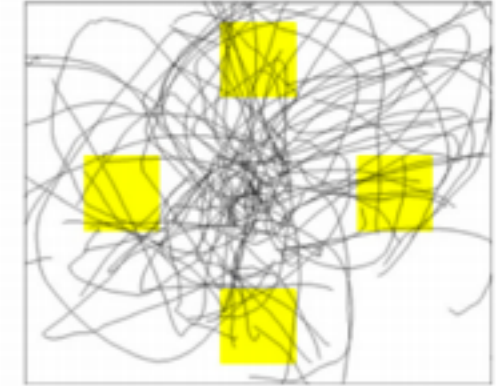
S3-254, 80 paths,  $N=46$



S3-261, 80 paths,  $N=80f$



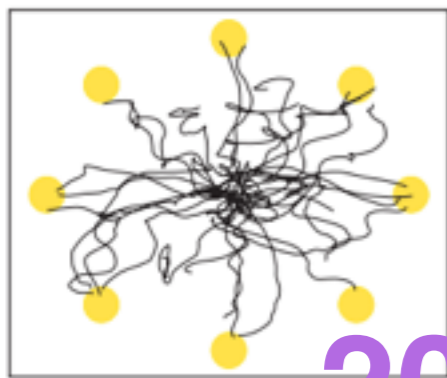
S3-280, 79 paths,  $N=13$



2010

Linear filter

31 paths, 25 units

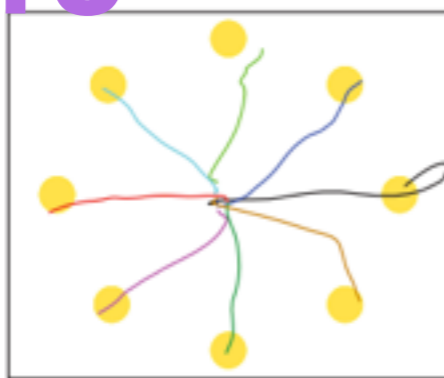
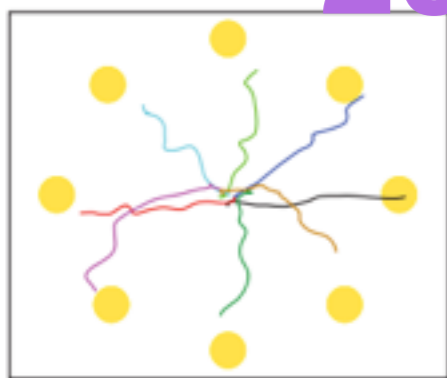


Kalman filter

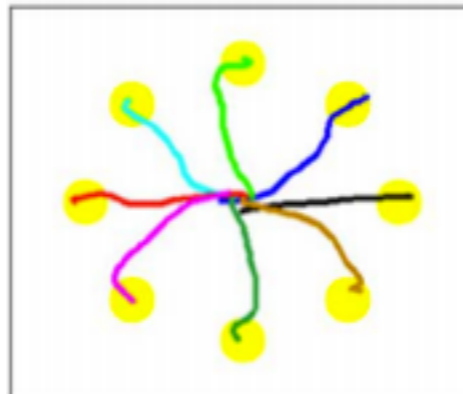
40 paths, 25 units



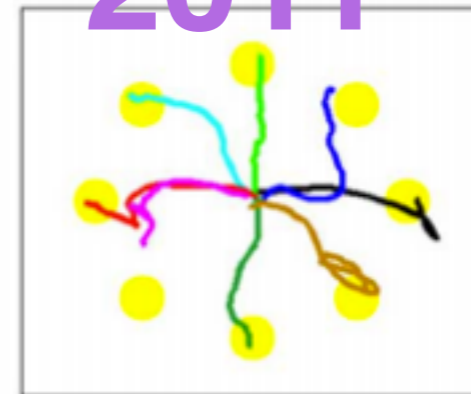
2013



C. S3, Day 303,  $N = 57$ ,  $n = 51$



D. S3, Day 464,  $N = 28$ ,  $n = 33$



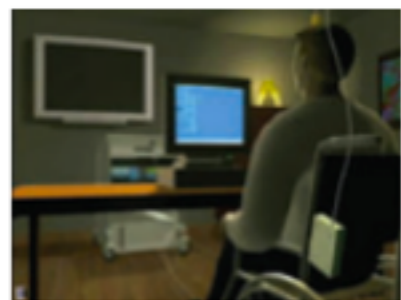
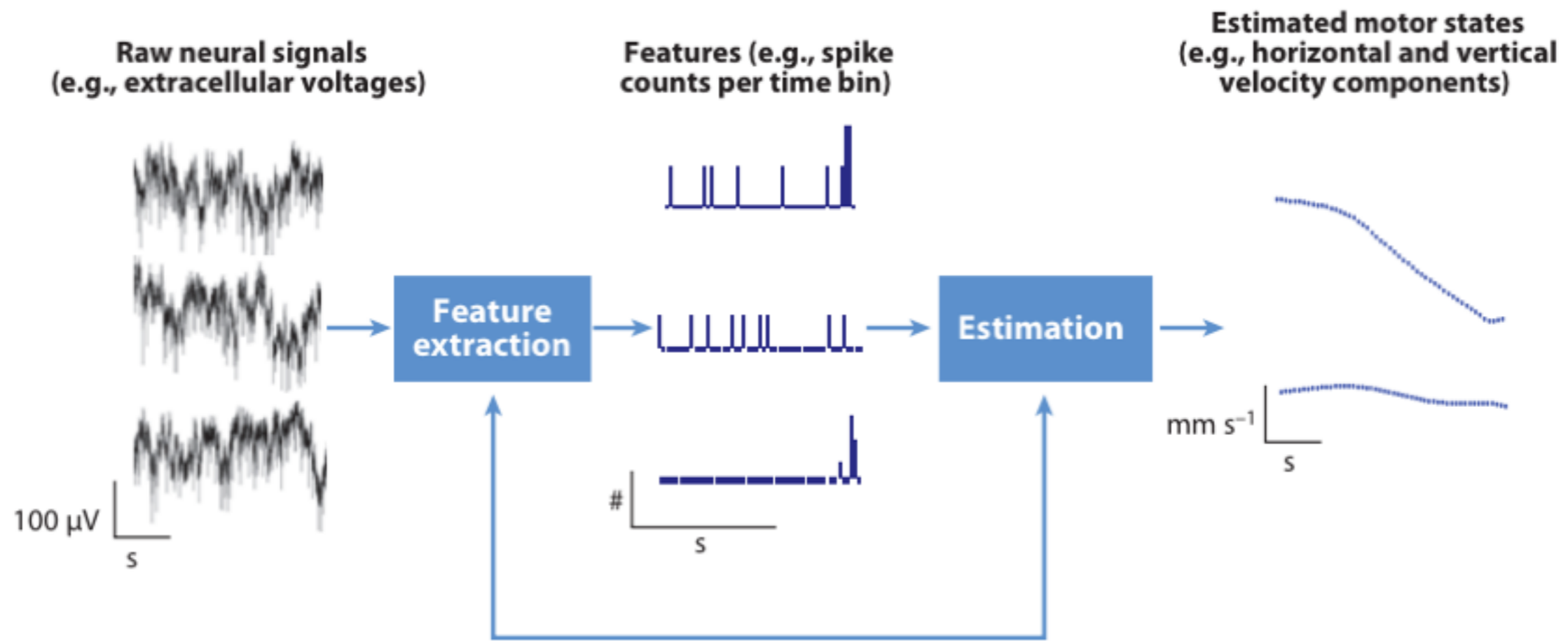
2011

E. A1, Day 231,  $N = 86$ ,  $n = 19$

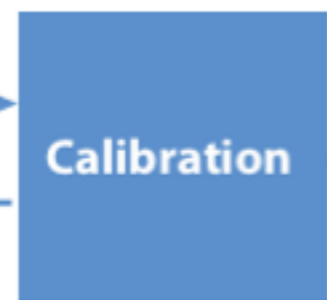


The **steady-state Kalman filter** significantly **increases** the computational **efficiency** for even relatively simple neural spiking data sets from a human NIS. <...> The **decoding complexity is reduced dramatically** by the SSKF, resulting in approximately seven-fold reduction in the execution time for decoding a typical neuronal firing rate signal.

# Summary



**Calibration trials**  
(e.g., intended motor states known)



Kalman filter  
A, H, Q, W matrices

**Estimated model parameters**  
(e.g., coefficients)

