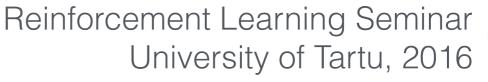




David Silver et al. from Google DeepMind

Mastering the game of Go with deep
neural networks and tree search

Article overview by Ilya Kuzovkin

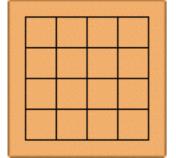


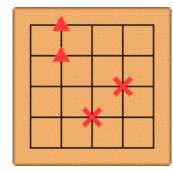




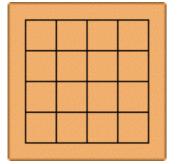
THE GAME OF GO

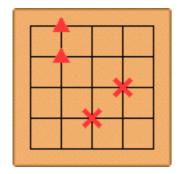
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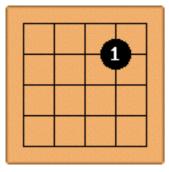


BOARD

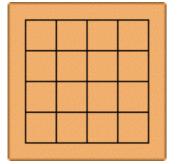


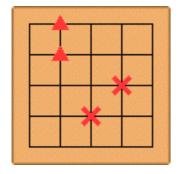


STONES

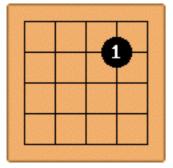


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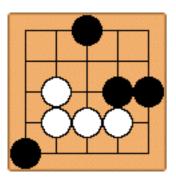




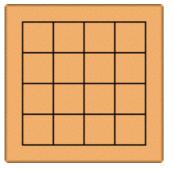
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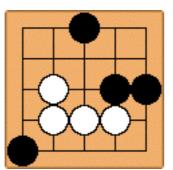
GROUPS

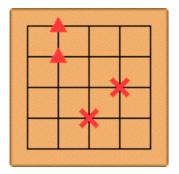


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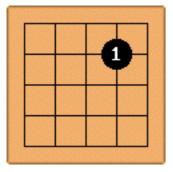


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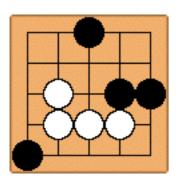




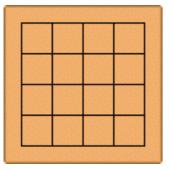
STONES



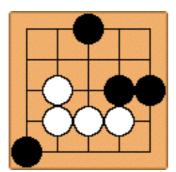
GROUPS



Board



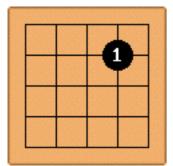
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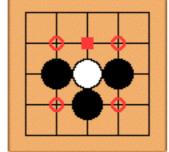


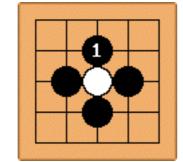
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Capture

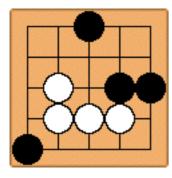
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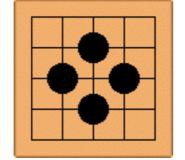




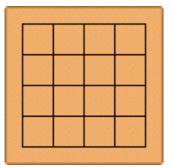


GROUPS

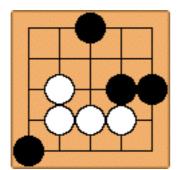




BOARD



LIBERTIES



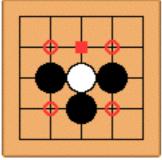
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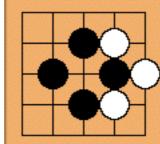
STONES

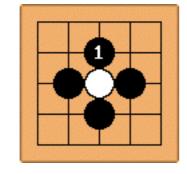
Capture

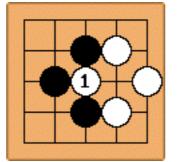


Ko

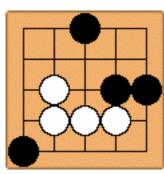


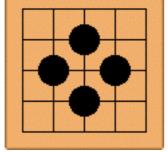


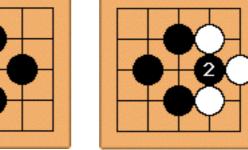




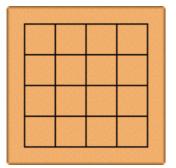
GROUPS



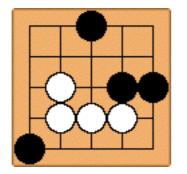




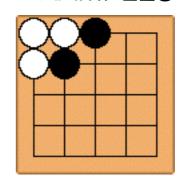
Board

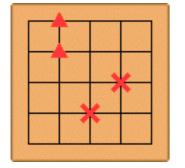


LIBERTIES



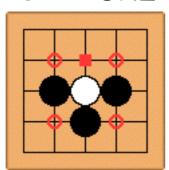
EXAMPLES



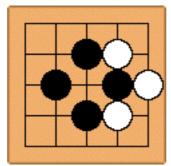


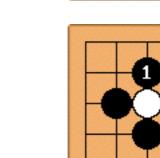
STONES

CAPTURE

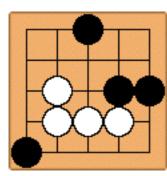


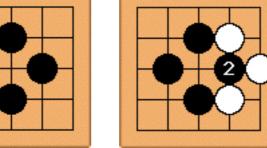
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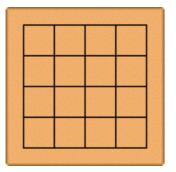


GROUPS

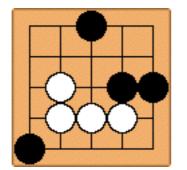




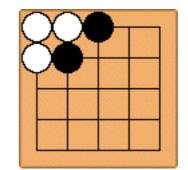
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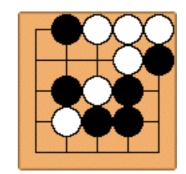


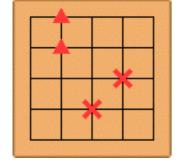
LIBERTIES



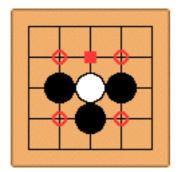
EXAMPLES



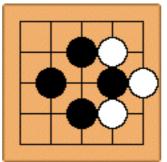




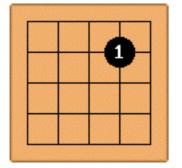
CAPTURE

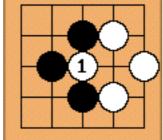


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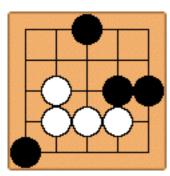


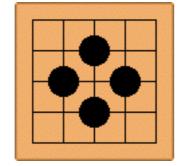
STONES

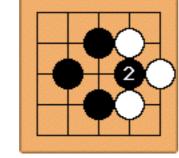




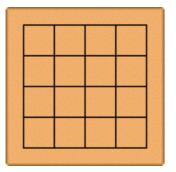
GROUPS



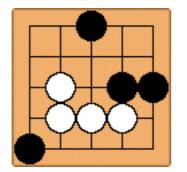




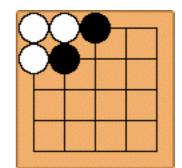
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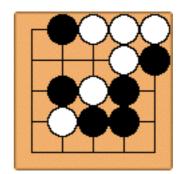


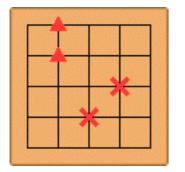
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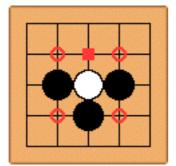
EXAMPLES



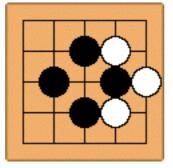




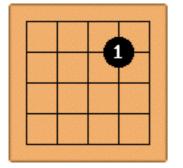
CAPTURE



Ko



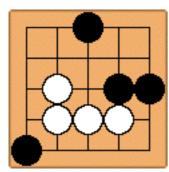
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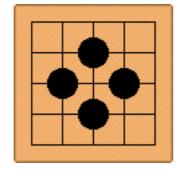


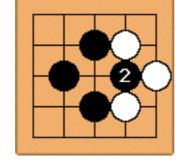
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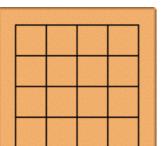
GROUPS

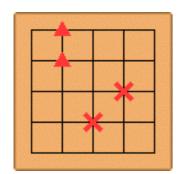




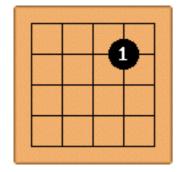


Board

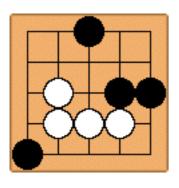




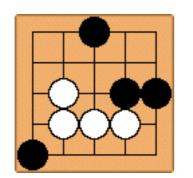
STONES



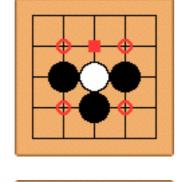
GROUPS

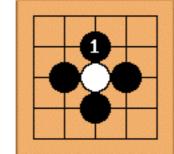


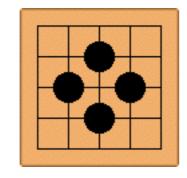
LIBERTIES



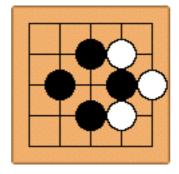
CAPTURE

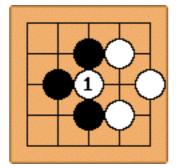


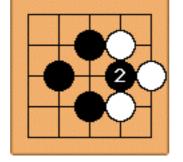




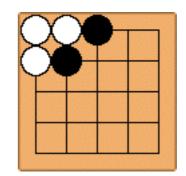
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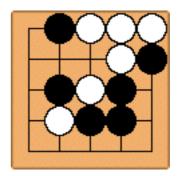


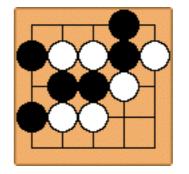




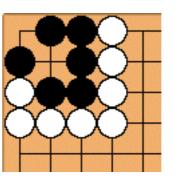
EXAMPLES



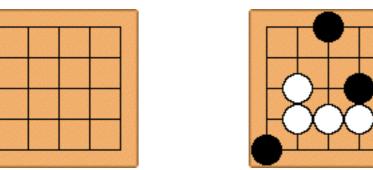


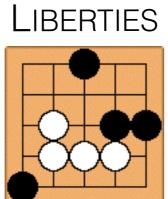


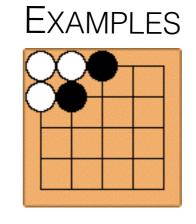
Two eyes

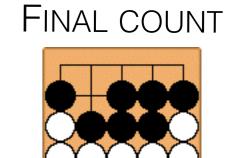


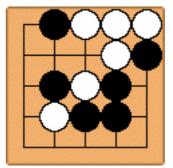
Board

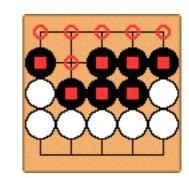


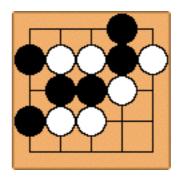


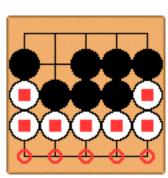




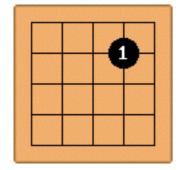




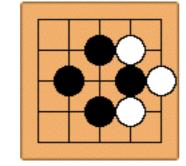




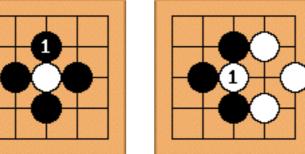
STONES



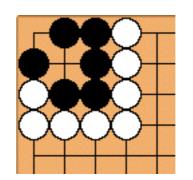
CAPTURE



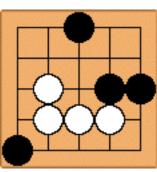
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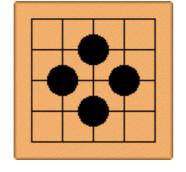


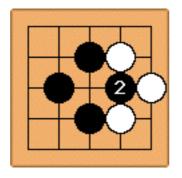




GROUPS









TRAINING THE BUILDING BLOCKS

SUPERVISED CLASSIFICATION

REINFORCEMENT

SUPERVISED REGRESSION

Supervised policy network $p_{\sigma}(a|s)$

Reinforcement policy network

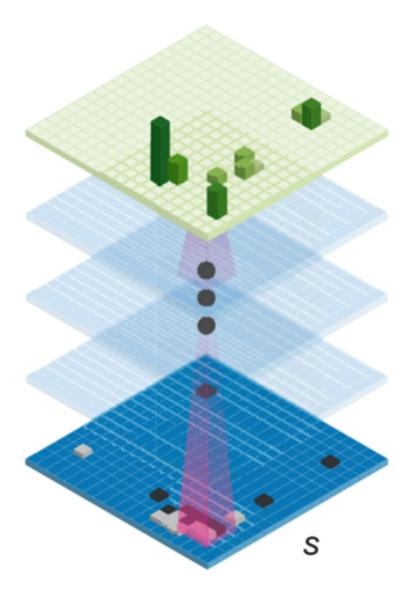
 $p_{\rho}(a|s)$

Value network $v_{ heta}(s)$

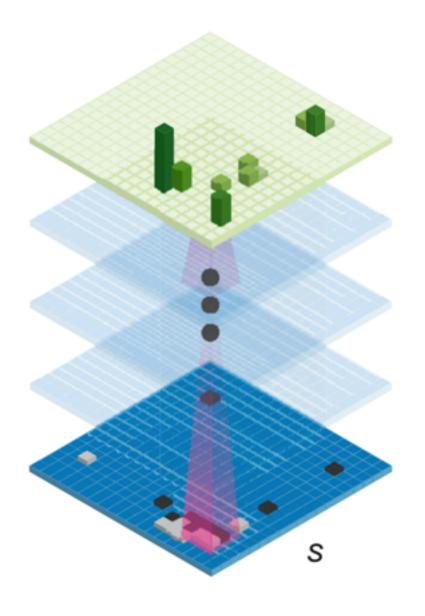
Rollout policy network $p_{\pi}(a|s)$

Tree policy network $p_{ au}(a|s)$

$$p_{\sigma\!/\!
ho}$$
 (a \mid s)



$$p_{\sigma/\rho}$$
 (a|s)



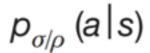
Softmax

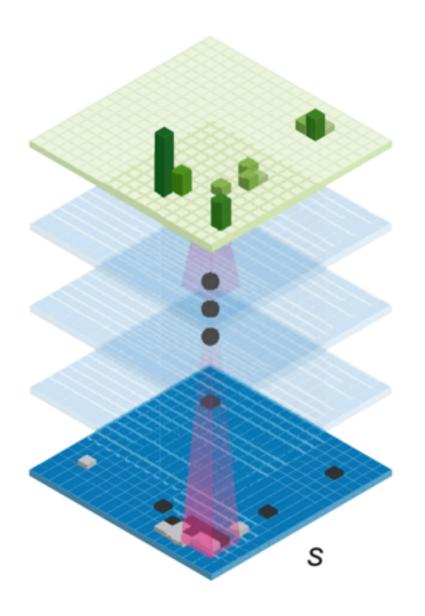
1 convolutional layer 1x1 ReLU

11 convolutional layers 3x3 with k=192 filters, ReLU

1 convolutional layer 5x5 with k=192 filters, ReLU

• 29.4M positions from games between 6 to 9 dan players





Softmax

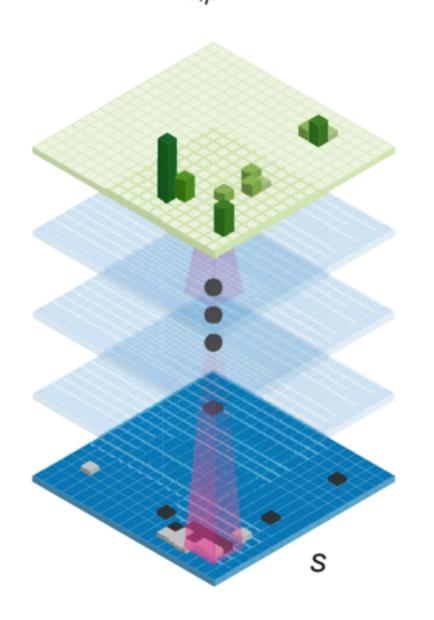
1 convolutional layer 1x1 ReLU

11 convolutional layers 3x3 with k=192 filters, ReLU

1 convolutional layer 5x5 with k=192 filters, ReLU

• 29.4M positions from games between 6 to 9 *dan* players

$$p_{\sigma/\rho}$$
 (a|s)



$$\Delta \sigma = \frac{\alpha}{m} \sum_{k=1}^{m} \frac{\partial \log p_{\sigma}(a^k | s^k)}{\partial \sigma}$$

Softmax

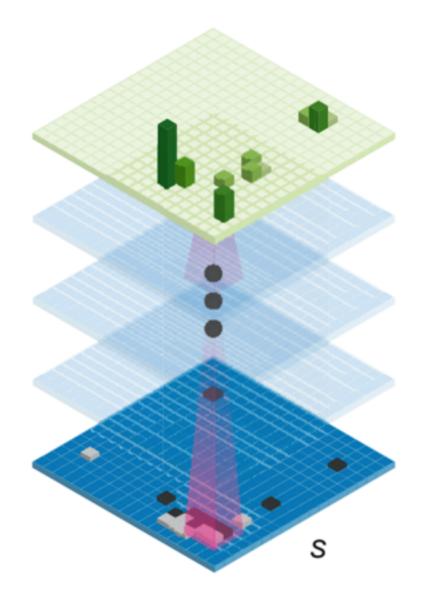
1 convolutional layer 1x1 ReLU

11 convolutional layers 3x3 with k=192 filters, ReLU

1 convolutional layer 5x5 with k=192 filters, ReLU

- stochastic gradient ascent
- learning rate α = 0.003, halved every 80M steps
- batch size m = 16
- 3 weeks on 50 GPUs to make 340M steps

$$p_{\sigma/\rho}$$
 (a|s)



- 29.4M positions from games between 6 to 9 dan players
- Augmented: 8 reflections/rotations
- Test set (1M) accuracy: 57.0%
- 3 ms to select an action

$$\Delta \sigma = \frac{\alpha}{m} \sum_{k=1}^{m} \frac{\partial \log p_{\sigma}(a^k | s^k)}{\partial \sigma}$$

Softmax

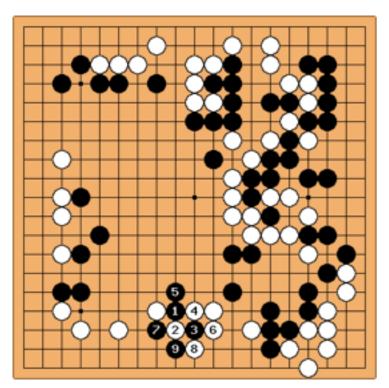
1 convolutional layer 1x1 ReLU

11 convolutional layers 3x3 with k=192 filters, ReLU

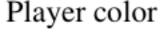
1 convolutional layer 5x5 with k=192 filters, ReLU

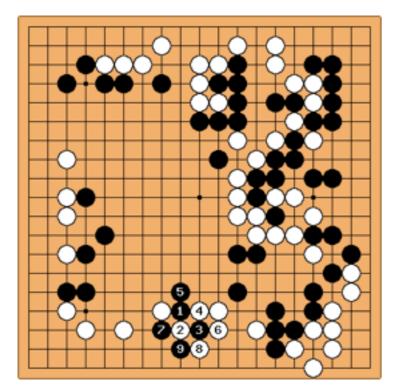
- stochastic gradient ascent
- learning rate α = 0.003, halved every 80M steps
- batch size m = 16
- 3 weeks on 50 GPUs to make 340M steps

Feature	# of planes	Description
Stone colour	3	Player stone / opponent stone / empty
Ones	1	A constant plane filled with 1
Turns since	8	How many turns since a move was played
Liberties	8	Number of liberties (empty adjacent points)
Capture size	8	How many opponent stones would be captured
Self-atari size	8	How many of own stones would be captured
Liberties after move	8	Number of liberties after this move is played
Ladder capture	1	Whether a move at this point is a successful ladder capture
Ladder escape	1	Whether a move at this point is a successful ladder escape
Sensibleness	1	Whether a move is legal and does not fill its own eyes
Zeros	1	A constant plane filled with 0
Player color	1	Whether current player is black



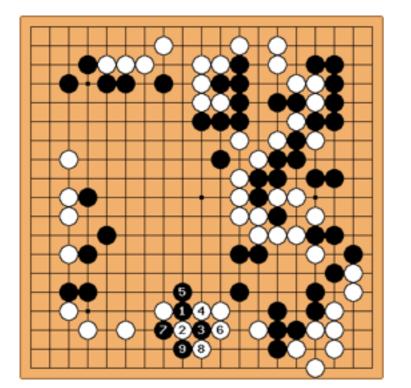
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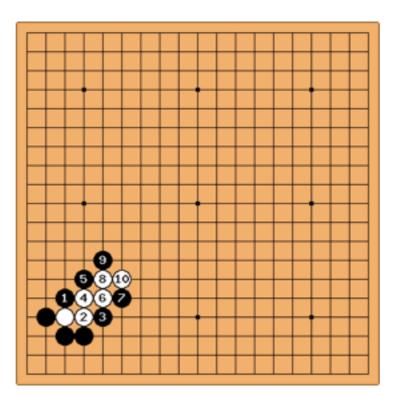


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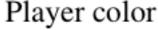
Player color

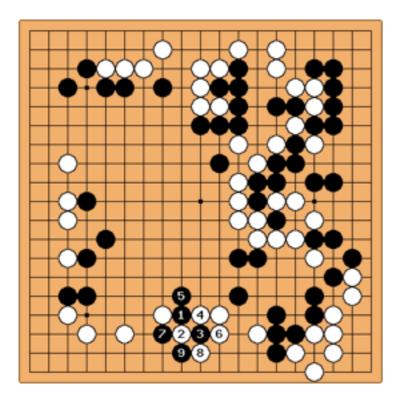


Whether current player is black

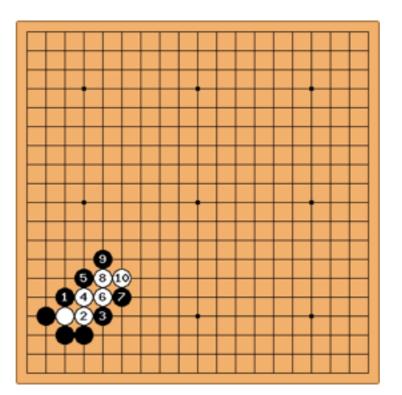


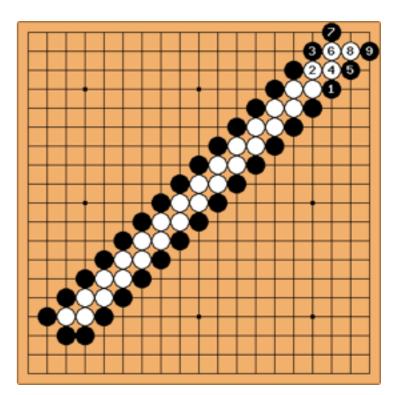
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1 Whether current player is black

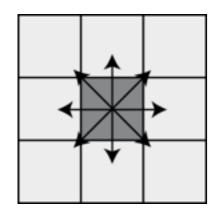


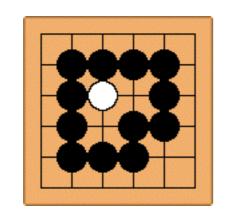


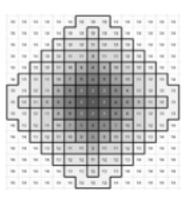
- Supervised same data as $p_{\sigma}(a|s)$
- Less accurate: 24.2% (vs. 57.0%)
- Faster: 2µs per action (1500 times)
- Just a linear model with softmax

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Feature	# of patterns	Description
Response	1	Whether move matches one or more response pattern features
Save atari	1	Move saves stone(s) from capture
Neighbour	8	Move is 8-connected to previous move
Nakade	8192	Move matches a nakade pattern at captured stone
Response pattern	32207	Move matches 12-point diamond pattern near previous move
Non-response pattern	69338	Move matches 3×3 pattern around move



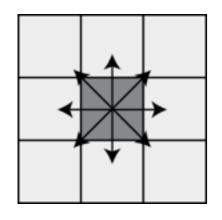


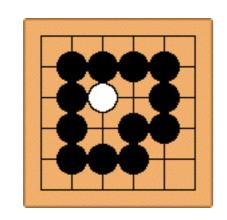


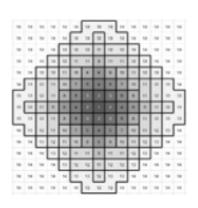
Tree policy $p_{\tau}(a|s)$

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Self-atari	1	Move allows stones to be captured
Last move distance	34	Manhattan distance to previous two moves
Non-response pattern	32207	Move matches 12-point diamond pattern centred around move





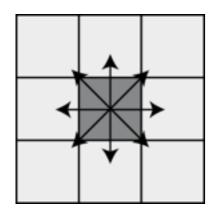


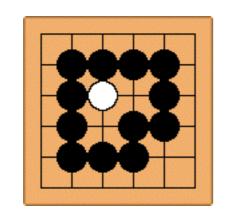
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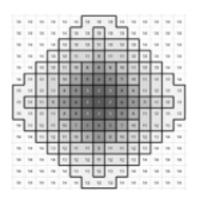
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•	"similar to the rollout policy but
	with more features"

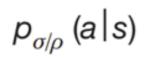
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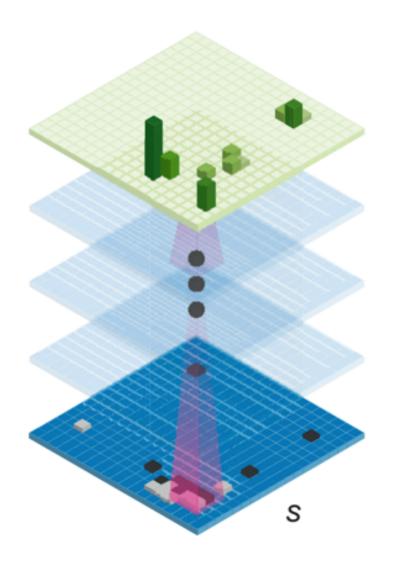






Reinforcement policy network $p_{\rho}(a|s)$

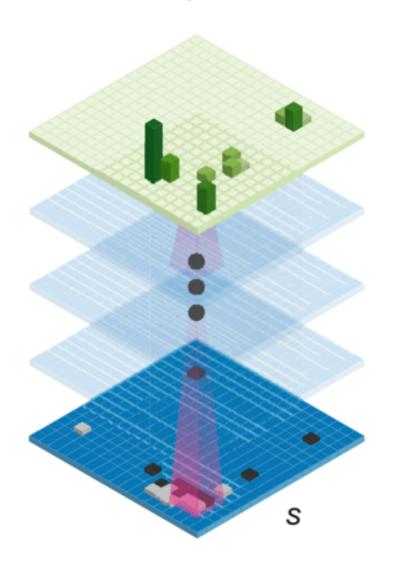




Same architecture Weights ρ are initialized with σ

Reinforcement policy network $p_{\rho}(a|s)$

$$p_{\sigma/\rho}$$
 (a|s)

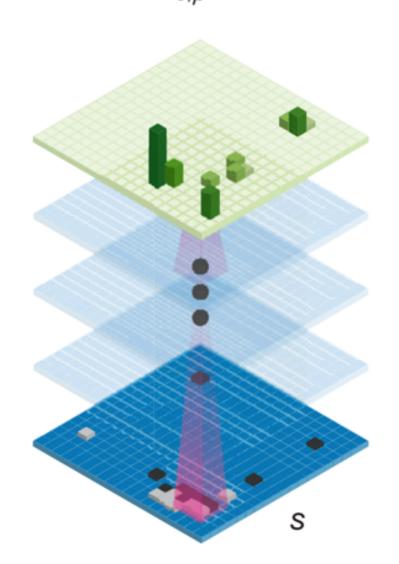


Same architecture Weights ρ are initialized with σ

Self-play: current network vs.
 randomized pool of previous versions

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Same architecture Weights ρ are initialized with σ

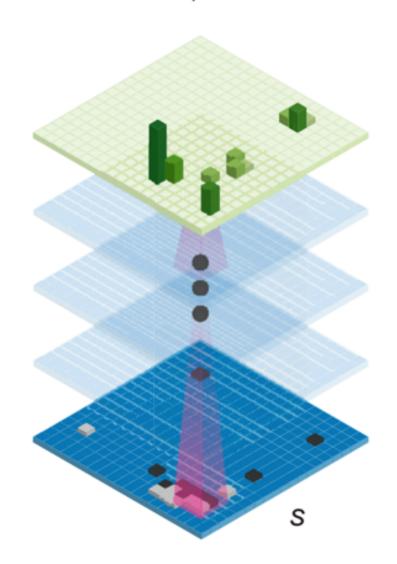
Self-play: current network vs.
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$$\Delta \rho = \frac{\alpha}{n} \sum_{i=1}^{n} \sum_{t=1}^{T^i} \frac{\partial \log p_{\rho}(a_t^i | s_t^i)}{\partial \rho} (z_t^i - \nu(s_t^i))$$

• Play a game until the end, get the reward $z_t = \pm r(s_T) = \pm 1$

$$p_{\rho}(a|s)$$

$$p_{\sigma/\rho}$$
 (a|s)



Same architecture Weights ρ are initialized with σ

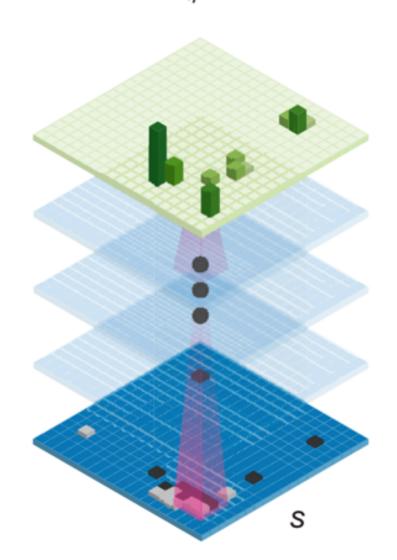
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 (a|s)



Same architecture Weights ρ are initialized with σ

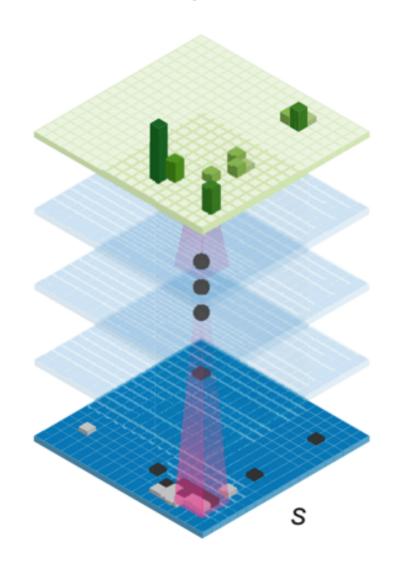
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- Set $z_t^\imath=z_t$ and play the same game again, this time updating the network parameters at each time step t
- $v(s_t^i) = \dots$
 - 0 "on the first pass through the training pipeline"
 - $v_{ heta}(s_t^i)$ "on the second pass"

$$p_{\rho}(a|s)$$

$$p_{\sigma/\rho}$$
 (a|s)



Same architecture Weights ρ are initialized with σ

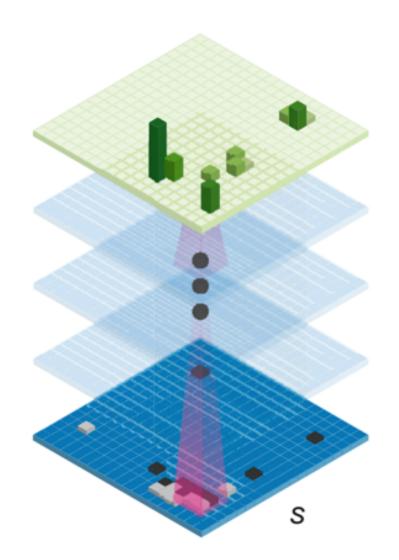
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- batch size n = 128 games
- 10,000 batches
- One day on 50 GPUs

$$p_{\rho}(a|s)$$

$$p_{\sigma/\rho}$$
 (a|s)



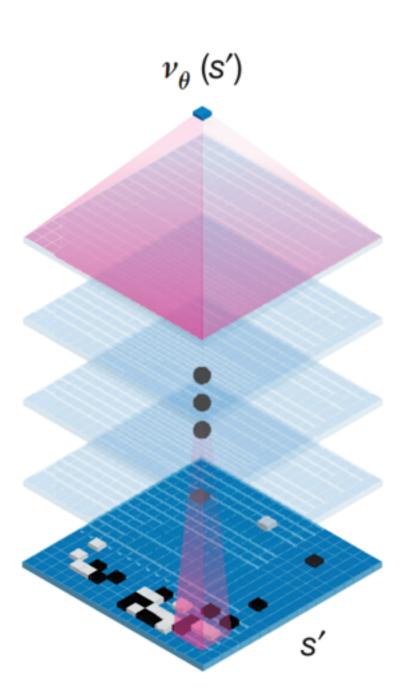
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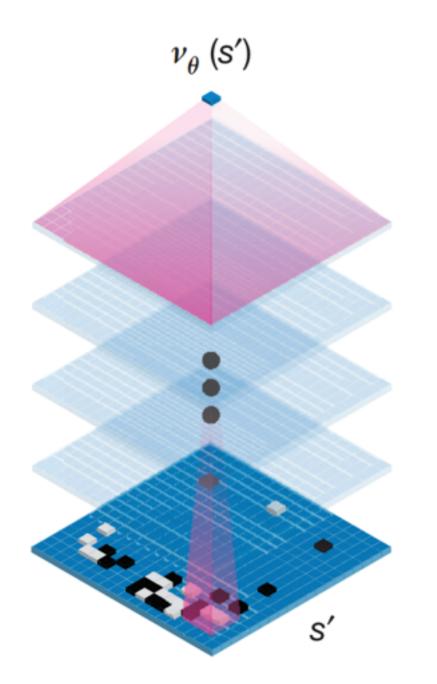
- Self-play: current network vs.
 randomized pool of previous versions
- 80% wins against Supervised Network
- 85% wins against *Pachi* (no search yet!)
- 3 ms to select an action

$$\Delta \rho = \frac{\alpha}{n} \sum_{i=1}^{n} \sum_{t=1}^{T^i} \frac{\partial \log p_{\rho}(a_t^i | s_t^i)}{\partial \rho} (z_t^i - \nu(s_t^i))$$

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Value network $v_{ heta}(s)$





Fully connected layer 1 tanh unit

Fully connected layer 256 ReLU units

1 convolutional layer 1x1 ReLU

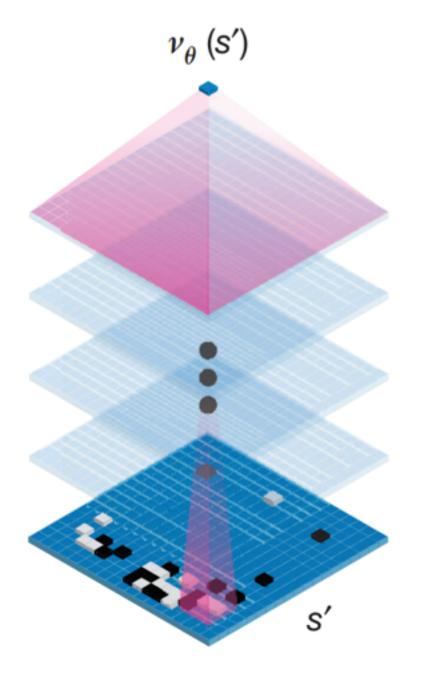
11 convolutional layers 3x3 with k=192 filters, ReLU

1 convolutional layer 5x5 with k=192 filters, ReLU

Evaluate the value of the position s under policy p:

$$v^p(s) = \mathbb{E}[z_t|s_t = s, a_{t...T} \sim p]$$

• Double approximation $\nu_{\theta}(s) \approx \nu^{p_{\rho}}(s) \approx \nu^{*}(s)$



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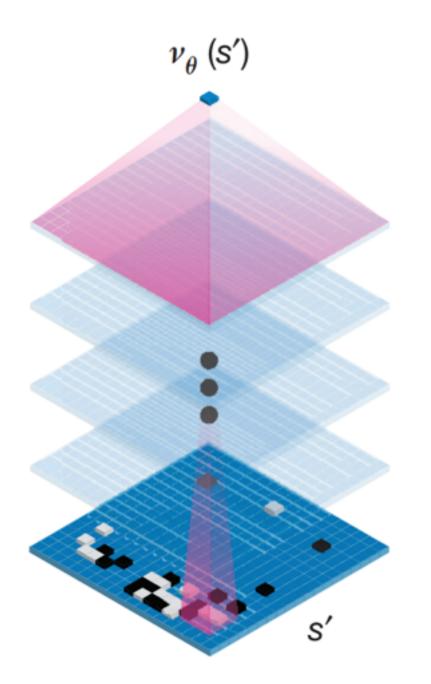
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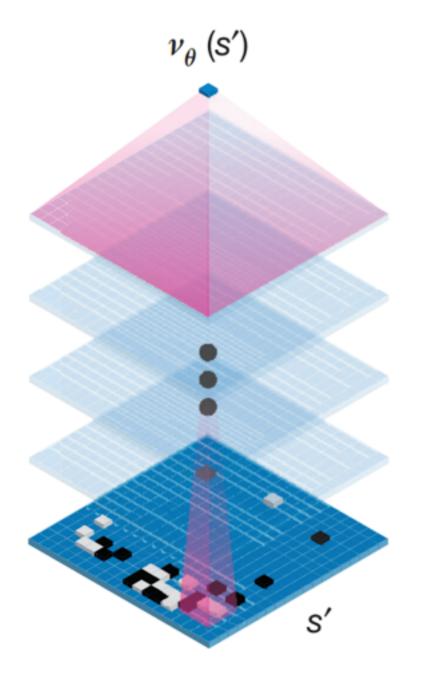
19 x 19 x 49 input

 Stochastic gradient descent to minimize MSE

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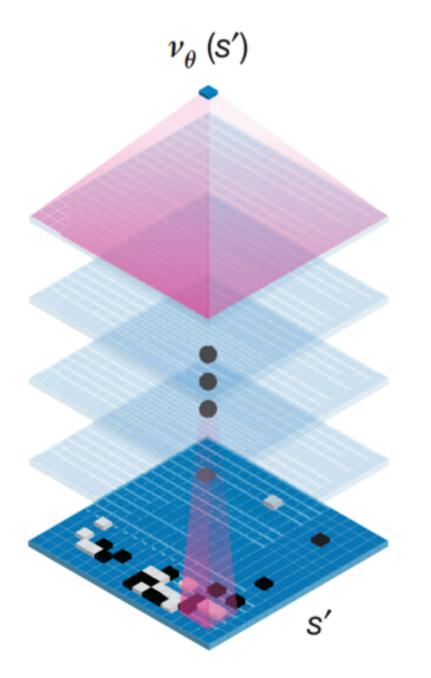
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- Stochastic gradient descent to minimize MSE
- Train on 30M state-outcome pairs (s,z), each from a unique game generated by self-play:

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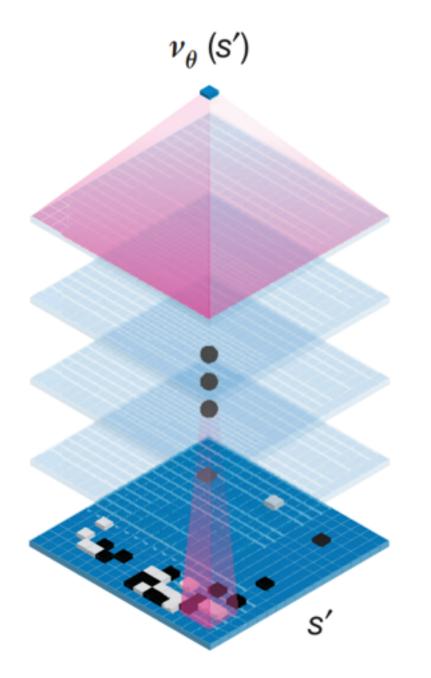
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 - choose a random time step u
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 - sample t=u+1...T from RL policy and get game outcome z
 - ullet add (s_u,z_u) pair to the training set



$$v^p(s) = \mathbb{E}[z_t|s_t = s, a_{t...T} \sim p]$$

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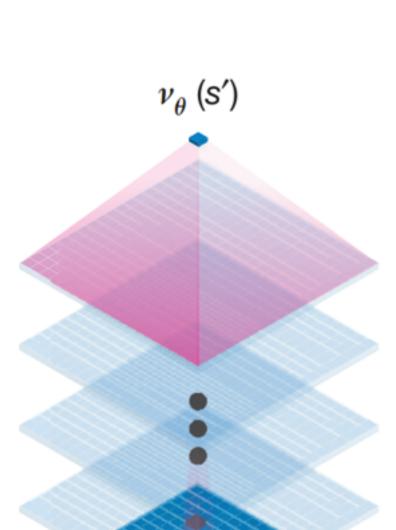
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- One week on 50 GPUs to train on 50M batches of size m=32



$$v^p(s) = \mathbb{E}[z_t|s_t = s, a_{t...T} \sim p]$$

- Double approximation $v_{\theta}(s) \approx v^{p_{\rho}}(s) \approx v^{*}(s)$
- MSE on the test set: 0.234
- Close to MC estimation from RL policy; 15,000 faster

$$\Delta \theta = \frac{\alpha}{m} \sum_{k=1}^{m} (z^k - \nu_{\theta}(s^k)) \frac{\partial \nu_{\theta}(s^k)}{\partial \theta}$$



Fully connected layer 1 tanh unit

Fully connected layer 256 ReLU units

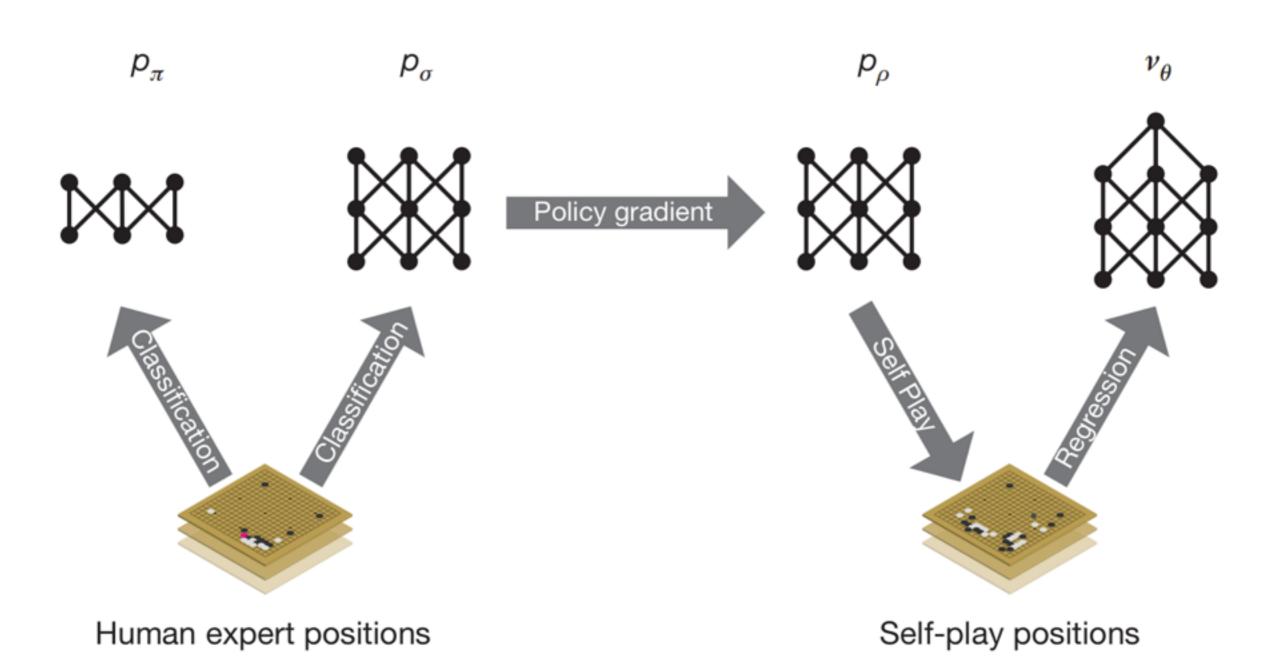
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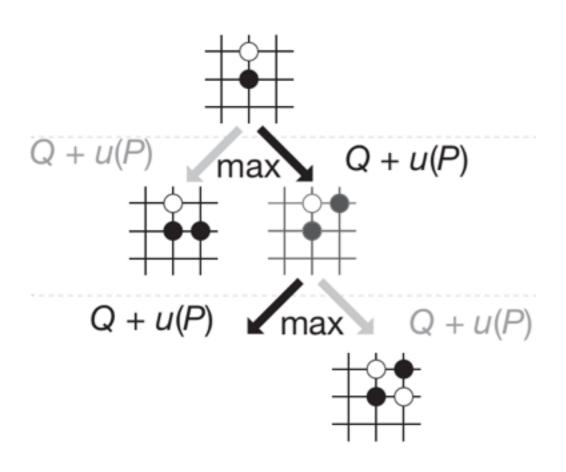
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Value network

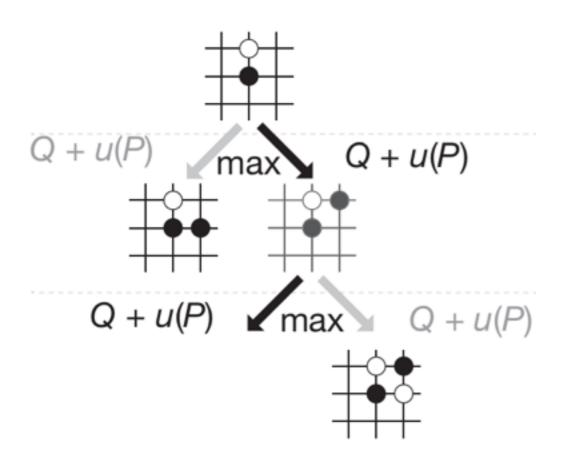




Selection



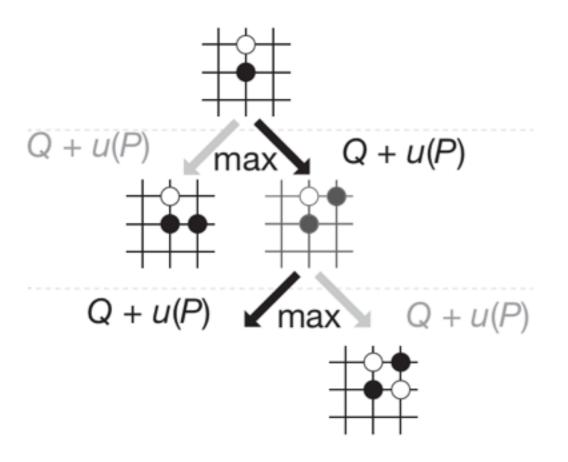
Selection



Each node *s* has edges (*s*, *a*) for all legal actions and stores statistics:

 $\{P(s,a),\ N_{v}(s,a),\ N_{r}(s,a),\ W_{v}(s,a),\ W_{r}(s,a),\ Q(s,a)\}$ Prior Number of evaluations Number of rollouts MC value estimate Rollout value Combined mean action value

Selection



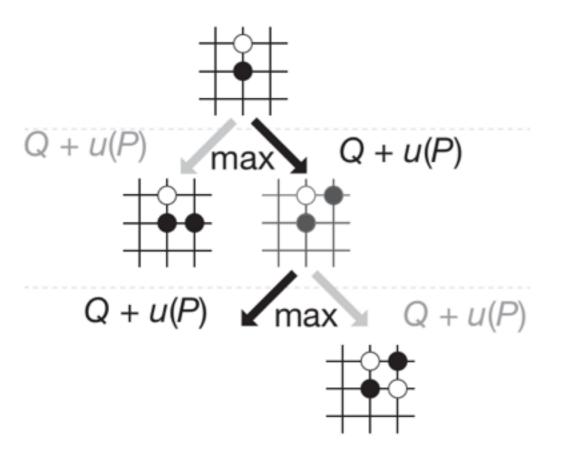
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 Prior Number of evaluations Number of evaluations rollouts Stimate St

Simulation starts at the root and stops at time L, when a leaf (unexplored state) is found. $a_t = \operatorname{argmax}_a (Q(s_t, a) + u(s_t, a))$

$$u(s,a) \propto \frac{P(s,a)}{1 + N(s,a)}$$

Selection



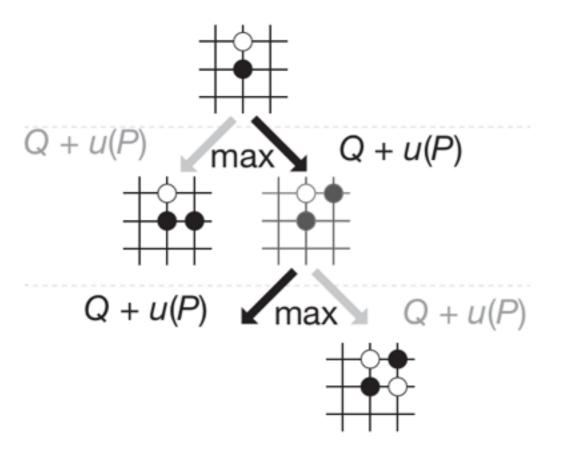
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$$\{P(s,a),\ N_{v}(s,a),\ N_{r}(s,a),\ W_{v}(s,a),\ W_{r}(s,a),\ Q(s,a)\}$$
 Prior Number of evaluations Number of evaluations rollouts action value estimate estimate value

Simulation starts at the root and stops at time L, when a leaf (unexplored state) is found. $a_t = \operatorname{argmax}_a (Q(s_t, a) + u(s_t, a))$ $u(s, a) \propto \frac{P(s, a)}{1 + N(s, a)}$

Position s_L is added to evaluation queue.

Selection



Each node s has edges (s, a) for all legal actions and stores statistics:

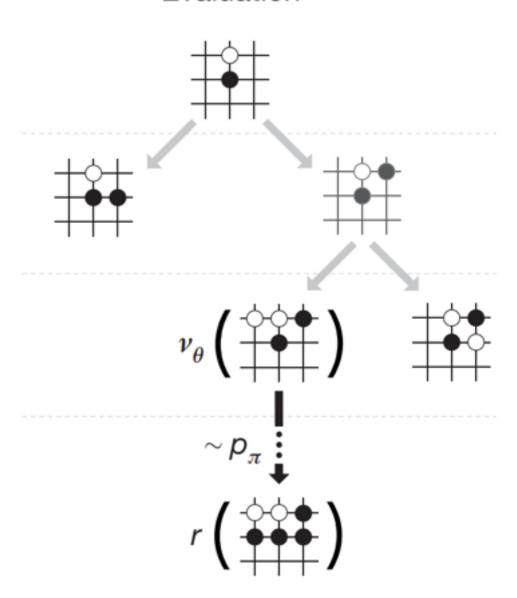
$$\{P(s,a),\ N_{v}(s,a),\ N_{r}(s,a),\ W_{v}(s,a),\ W_{r}(s,a),\ Q(s,a)\}$$
 Prior Number of evaluations Number of evaluations rollouts action value estimate estimate value

Simulation starts at the root and stops at time L, when a leaf (unexplored state) is found. $a_t = \operatorname{argmax}_a \left(Q(s_t, a) + u(s_t, a) \right)$ $u(s, a) \propto \frac{P(s, a)}{1 + N(s, a)}$

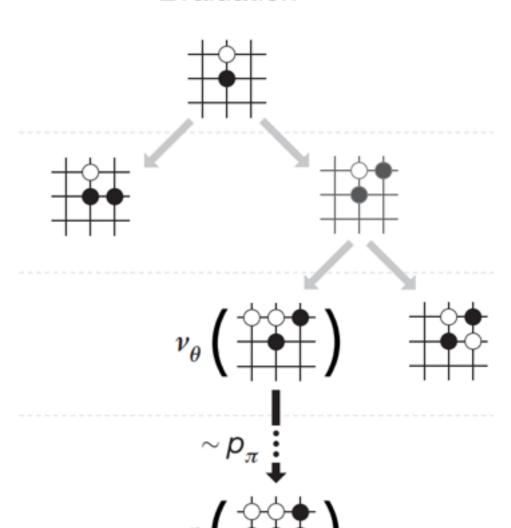
Position s_L is added to evaluation queue.

Bunch of nodes selected for evaluation...

Evaluation

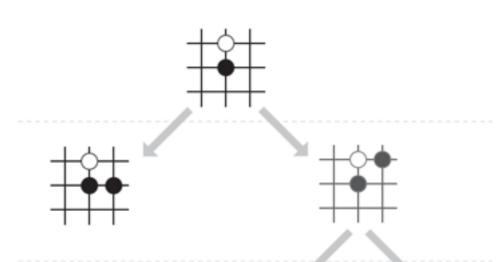


Evaluation



Node s is evaluated using the value network to obtain $v_{ heta}(s)$

Evaluation



Node *s* is evaluated using the value network to obtain

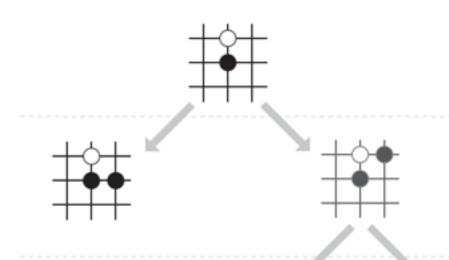
$$v_{\theta}(s)$$

 $v_{\theta} \left(\begin{array}{c} \downarrow \downarrow \downarrow \downarrow \\ \downarrow \downarrow \downarrow \downarrow \\ \\ \sim \rho_{\pi} \end{array} \right)$

and using rollout simulation with policy p_{π} till the end of each simulated game to get the final game score.

$$z_t = \pm r(s_T)$$

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$$v_{\theta}(s)$$

 $\nu_{\theta} \left(\begin{array}{c} \uparrow \downarrow \downarrow \downarrow \downarrow \\ \downarrow \downarrow \downarrow \downarrow \\ \\ \sim \rho_{\pi} \\ \downarrow \downarrow \\ \end{array} \right)$

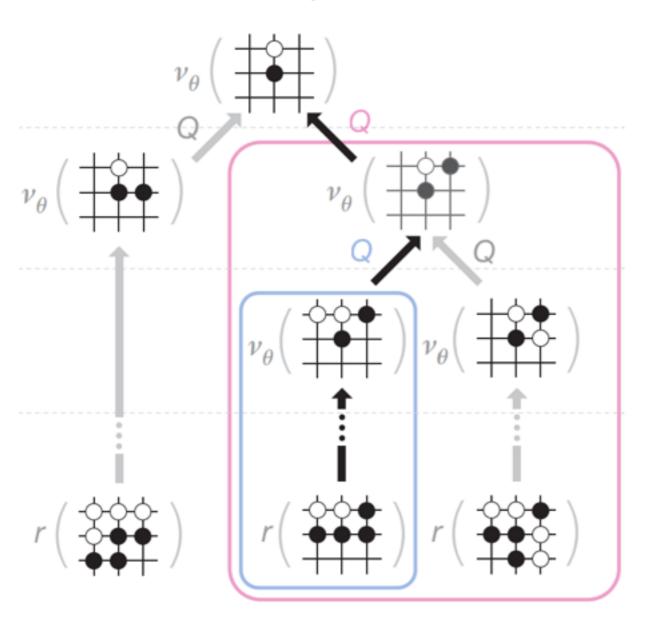
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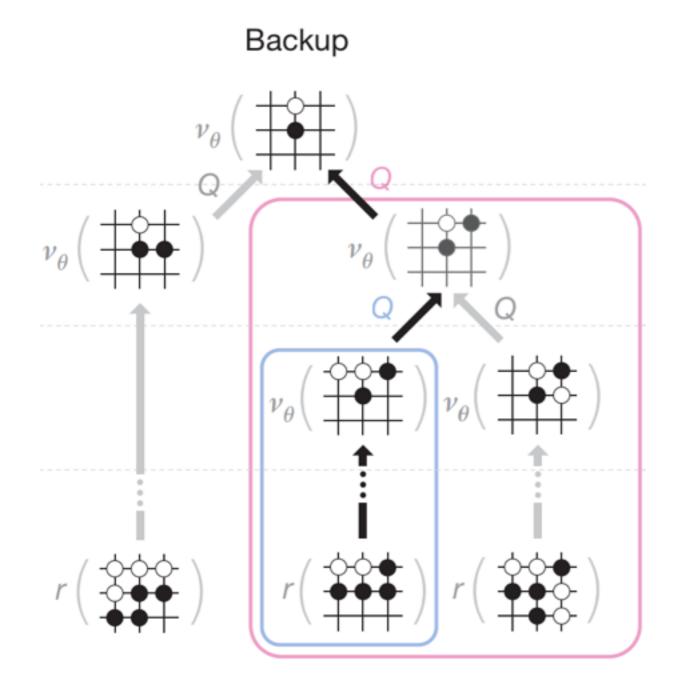
$$z_t = \pm r(s_T)$$

r († † †)

Each leaf is evaluated, we are ready to propagate updates

Backup



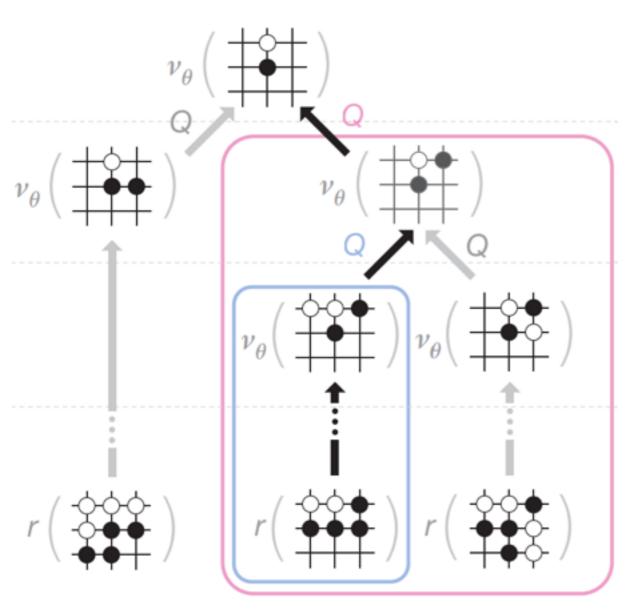


Statistics along the paths of each simulation are updated during the backward pass though t < L

$$W_{\nu}(s_t, a_t) \leftarrow W_{\nu}(s_t, a_t) + \nu_{\theta}(s_L)$$

 $W_{r}(s_t, a_t) \leftarrow W_{r}(s_t, a_t) + z_t$





Statistics along the paths of each simulation are updated during the backward pass though t < L

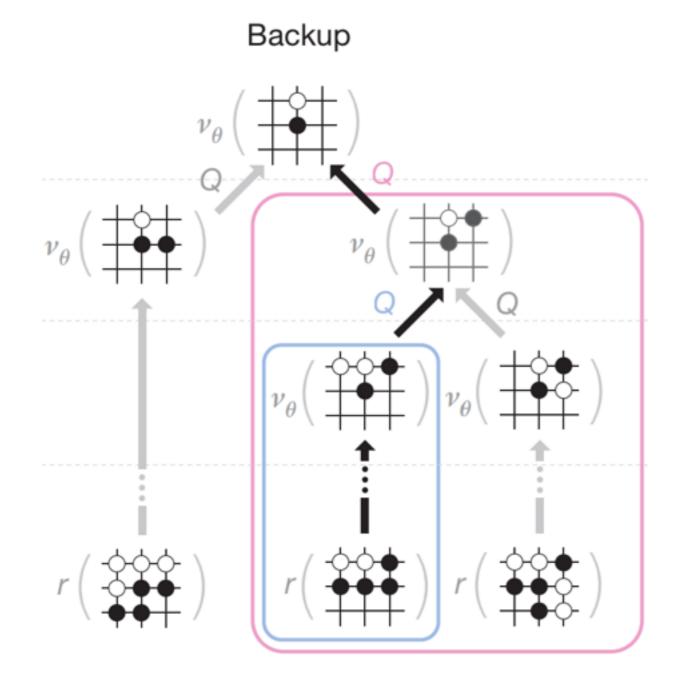
$$W_{\nu}(s_t, a_t) \leftarrow W_{\nu}(s_t, a_t) + \nu_{\theta}(s_L)$$

 $W_{r}(s_t, a_t) \leftarrow W_{r}(s_t, a_t) + z_t$

visits counts are updated as well

$$N_r(s_t, a_t) \leftarrow N_r(s_t, a_t) + 1$$

 $N_v(s_t, a_t) \leftarrow N_v(s_t, a_t) + 1$



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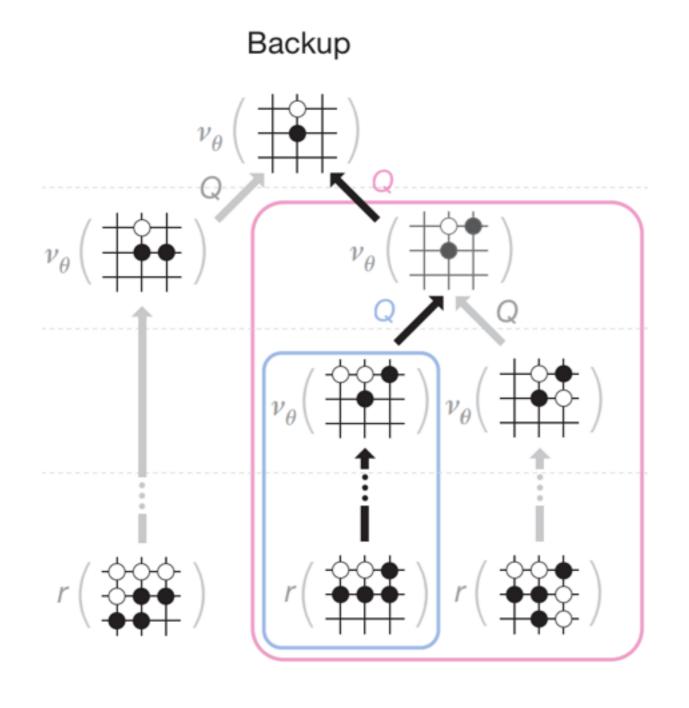
visits counts are updated as well

$$N_r(s_t, a_t) \leftarrow N_r(s_t, a_t) + 1$$

 $N_v(s_t, a_t) \leftarrow N_v(s_t, a_t) + 1$

Finally overall evaluation of each visited state-action edge is updated

$$Q(s, a) = (1 - \lambda) \frac{W_{\nu}(s, a)}{N_{\nu}(s, a)} + \lambda \frac{W_{r}(s, a)}{N_{r}(s, a)}$$



Statistics along the paths of each simulation are updated during the backward pass though t < L

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$$N_r(s_t, a_t) \leftarrow N_r(s_t, a_t) + 1$$

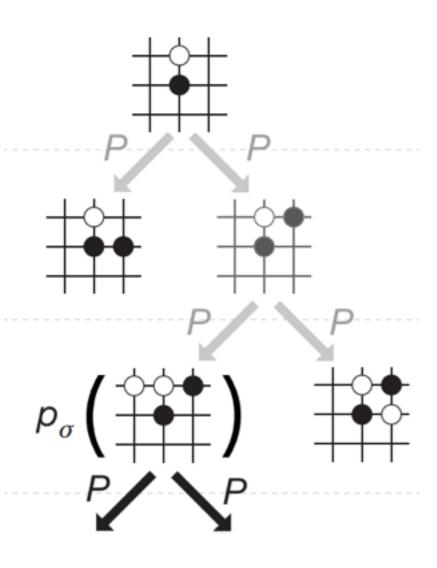
 $N_v(s_t, a_t) \leftarrow N_v(s_t, a_t) + 1$

Finally overall evaluation of each visited state-action edge is updated

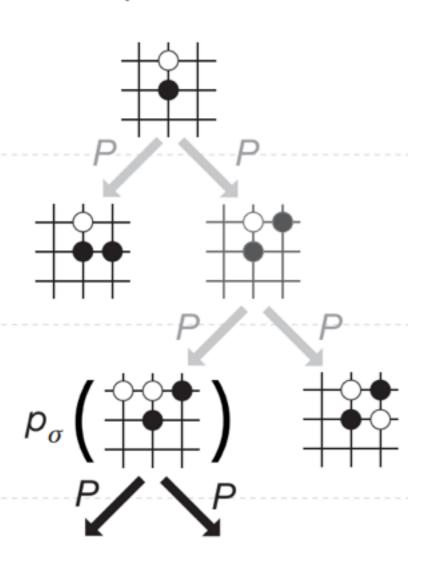
$$Q(s, a) = (1 - \lambda) \frac{W_{\nu}(s, a)}{N_{\nu}(s, a)} + \lambda \frac{W_{r}(s, a)}{N_{r}(s, a)}$$

Current tree is updated

Expansion

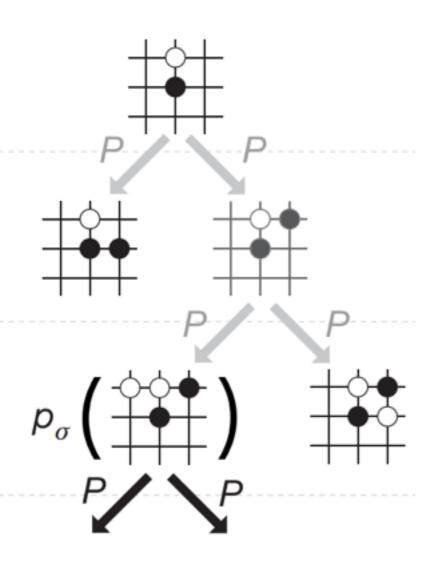


Expansion



Once an edge (s, a) is visited enough ($n_{\rm thr}$) times it is included into the tree with s'

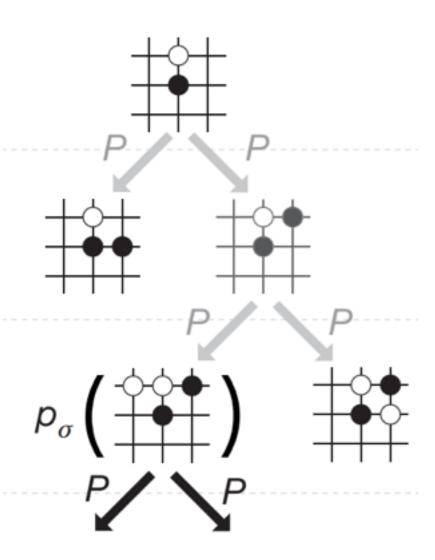
Expansion



Once an edge (s, a) is visited enough ($n_{\rm thr}$) times it is included into the tree with s'

It is initialized using the tree policy $p_{\tau}(a|s')$ to $\{N(s',a)=N_r(s',a)=0, W(s',a)=W_r(s',a)=0, P(s',a)=p_{\tau}(a|s')\}$ and updated with SL policy: $P(s',a)\leftarrow p_{\sigma}(a|s')$

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Tree is expanded, fully updated and ready for the next move!

